



## UNIT 6

# Probability

MODULE

12

### Experimental Probability



FL

7.SP.3.5, 7.SP.3.6,  
7.SP.3.7a, 7.SP.3.7b,  
7.SP.3.8, 7.SP.3.8a,  
7.SP.3.8b, 7.SP.3.8c

MODULE

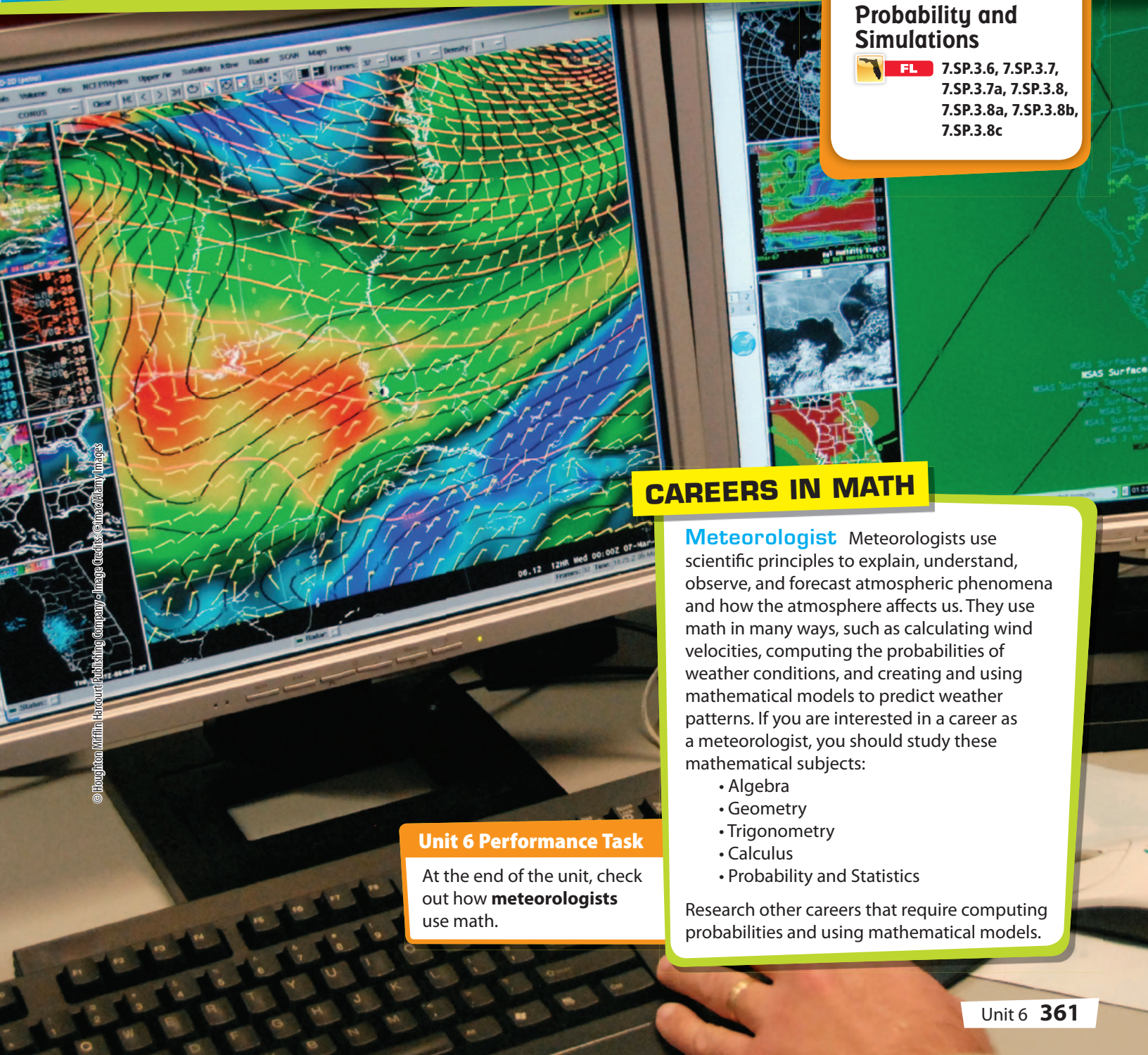
13

### Theoretical Probability and Simulations



FL

7.SP.3.6, 7.SP.3.7,  
7.SP.3.7a, 7.SP.3.8,  
7.SP.3.8a, 7.SP.3.8b,  
7.SP.3.8c



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## CAREERS IN MATH

**Meteorologist** Meteorologists use scientific principles to explain, understand, observe, and forecast atmospheric phenomena and how the atmosphere affects us. They use math in many ways, such as calculating wind velocities, computing the probabilities of weather conditions, and creating and using mathematical models to predict weather patterns. If you are interested in a career as a meteorologist, you should study these mathematical subjects:

- Algebra
- Geometry
- Trigonometry
- Calculus
- Probability and Statistics

Research other careers that require computing probabilities and using mathematical models.

### Unit 6 Performance Task

At the end of the unit, check out how **meteorologists** use math.

# Vocabulary Preview

Use the puzzle to preview key vocabulary from this unit. Unscramble the circled letters within found words to answer the riddle at the bottom of the page.

Y	R	F	S	P	D	A	U	L	B	S	T	Y	C	P
T	N	U	E	A	G	V	V	X	I	Y	R	C	A	R
N	N	B	A	H	M	C	P	M	F	U	I	Y	H	O
R	E	E	X	C	K	P	P	L	O	O	A	A	X	B
K	V	H	M	H	O	L	D	G	H	L	T	O	A	
Q	L	R	B	I	E	W	V	E	Z	V	I	W	S	B
L	F	E	B	E	R	X	E	X	S	I	F	Y	I	I
Q	V	I	V	T	N	E	M	E	L	P	M	O	C	L
Q	O	E	S	H	J	R	P	S	R	K	A	L	E	I
A	N	H	C	J	D	D	O	X	H	P	F	C	X	T
T	P	Z	K	V	Z	V	F	N	E	P	F	U	E	Y
U	V	K	S	I	M	U	L	A	T	I	O	N	P	J
K	X	O	L	P	M	O	M	S	U	Z	J	W	A	P
R	O	C	P	T	U	N	D	N	V	E	R	T	U	A
U	W	O	G	L	B	U	H	K	E	S	F	A	P	B

- An activity based on chance in which results are observed. (Lesson 12.1)
- The set of all outcomes that are not included in the event. (Lesson 12.1)
- Each observation of an experiment. (Lesson 12.1)
- A model of an experiment that would be difficult or too time-consuming to perform. (Lesson 12.2)
- Measures the likelihood that the event will occur. (Lesson 12.1)
- An event with only one outcome (2 words). (Lesson 12.2)
- A set of all possible outcomes for an event (2 words). (Lesson 12.1)

**Q:** Why was there little chance of success for the clumsy thieves?

**A:** Because they had low \_\_\_\_\_ - \_\_\_\_\_!

# Experimental Probability

MODULE



# 12



## ESSENTIAL QUESTION

How can you use experimental probability to solve real-world problems?

LESSON 12.1

### Probability

**FL** 7.SP.3.5, 7.SP.3.7a

LESSON 12.2

### Experimental Probability of Simple Events

**FL** 7.SP.3.6, 7.SP.3.7b

LESSON 12.3

### Experimental Probability of Compound Events

**FL** 7.SP.3.8, 7.SP.3.8a, 7.SP.3.8b, 7.SP.3.8c

LESSON 12.4

### Making Predictions with Experimental Probability

**FL** 7.SP.3.6



#### Real-World Video

Meteorologists use sophisticated equipment to gather data about the weather. Then they use experimental probability to forecast, or predict, what the weather conditions will be.

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# Are YOU Ready?

Complete these exercises to review skills you will need for this module.



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## Simplify Fractions

**EXAMPLE** Simplify  $\frac{12}{21}$ .

12: 1, 2, ③ 4, 6, 12    List all the factors of the numerator and denominator.  
21: 1, ③ 7, 21    Circle the greatest common factor (GCF).

$\frac{12 \div 3}{21 \div 3} = \frac{4}{7}$     Divide the numerator and denominator by the GCF.

Write each fraction in simplest form.

- $\frac{6}{10}$  \_\_\_\_\_
- $\frac{9}{15}$  \_\_\_\_\_
- $\frac{16}{24}$  \_\_\_\_\_
- $\frac{9}{36}$  \_\_\_\_\_
- $\frac{45}{54}$  \_\_\_\_\_
- $\frac{30}{42}$  \_\_\_\_\_
- $\frac{36}{60}$  \_\_\_\_\_
- $\frac{14}{42}$  \_\_\_\_\_

## Write Fractions as Decimals

**EXAMPLE**  $\frac{13}{25} \rightarrow$

$$\begin{array}{r} 0.52 \\ 25 \overline{)13.00} \\ \underline{-12.5} \phantom{0} \\ 50 \\ \underline{-50} \\ 0 \end{array}$$

Write the fraction as a division problem.  
Write a decimal point and a zero in the dividend.  
Place a decimal point in the quotient.  
Write more zeros in the dividend if necessary.

Write each fraction as a decimal.

- $\frac{3}{4}$  \_\_\_\_\_
- $\frac{7}{8}$  \_\_\_\_\_
- $\frac{3}{20}$  \_\_\_\_\_
- $\frac{19}{50}$  \_\_\_\_\_

## Percents and Decimals

**EXAMPLE**  $109\% = 100\% + 9\%$   
 $= \frac{100}{100} + \frac{9}{100}$   
 $= 1 + 0.09$   
 $= 1.09$

Write the percent as the sum of 1 whole and a percent remainder.  
Write the percents as fractions.  
Write the fractions as decimals.  
Simplify.

Write each percent as a decimal.

- 67% \_\_\_\_\_
- 31% \_\_\_\_\_
- 7% \_\_\_\_\_
- 146% \_\_\_\_\_

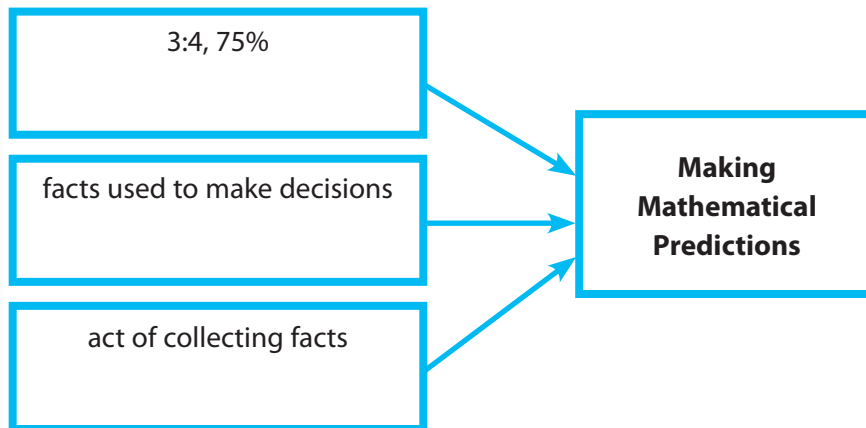
Write each decimal as a percent.

- 0.13 \_\_\_\_\_
- 0.55 \_\_\_\_\_
- 0.08 \_\_\_\_\_
- 1.16 \_\_\_\_\_

# Reading Start-Up

## Visualize Vocabulary

Use the ✓ words to complete the graphic. You can put more than one word in each box.



## Understand Vocabulary

Match the term on the left to the definition on the right.

- |                |   |
|----------------|---|
| 1. probability | A. Measures the likelihood that the event will occur. |
| 2. trial       | B. A set of one or more outcomes.                     |
| 3. event       | C. Each observation of an experiment.                 |

## Vocabulary

### Review Words

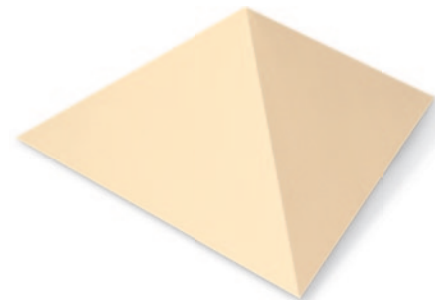
- ✓ data (*datos*)
- ✓ observation (*observación*)
- ✓ percent (*porcentaje*)
- ✓ ratio (*razón*)

### Preview Words

- complement (*complemento*)
- compound event (*suceso compuesto*)
- event (*suceso*)
- experiment (*experimento*)
- experimental probability (*probabilidad experimental*)
- outcome (*resultado*)
- probability (*probabilidad*)
- simple event (*suceso simple*)
- simulation (*simulación*)
- trial (*prueba*)

## Active Reading

**Pyramid** Before beginning the module, create a rectangular pyramid to help you organize what you learn. Label each side with one of the lesson titles from this module. As you study each lesson, write important ideas, such as vocabulary, properties, and formulas, on the appropriate side.





# Unpacking the Standards

Understanding the standards and the vocabulary terms in the standards will help you know exactly what you are expected to learn in this module.

**FL 7.SP.3.6**

Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability.

**Key Vocabulary****simple event** (*suceso simple*)

An event consisting of only one outcome.

**experimental probability** (*probabilidad experimental*)

The ratio of the number of times an event occurs to the total number of trials, or times that the activity is performed.

## What It Means to You

You will use experimental probabilities to make predictions and solve problems.

**UNPACKING EXAMPLE 7.SP.3.6**

Caitlyn finds that the experimental probability of her making a goal in hockey is 30%. Out of 500 attempts to make a goal, about how many could she predict she would make?

$$\frac{3}{10} \cdot 500 = x$$

$$150 = x$$

Caitlyn can predict that she will make about 150 of the 500 goals that she attempts.

**FL 7.SP.3.7b**

Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process.

**Key Vocabulary****sample space** (*espacio muestral*)

All possible outcomes of an experiment.

## What It Means to You

You will use data to determine experimental probabilities.

**UNPACKING EXAMPLE 7.SP.3.7b**

Anders buys a novelty coin that is weighted more heavily on one side. He flips the coin 60 times and a head comes up 36 times. Based on his results, what is the experimental probability of flipping a head?

$$\text{experimental probability} = \frac{\text{number of times event occurs}}{\text{total number of trials}}$$

$$= \frac{36}{60} = \frac{3}{5}$$

The experimental probability of flipping a head is  $\frac{3}{5}$ .



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# LESSON 12.1 Probability

 **FL** 7.SP.3.5

Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. ... Also 7.SP.3.7a



## ESSENTIAL QUESTION

How can you describe the likelihood of an event?

### EXPLORE ACTIVITY

 **FL** 7.SP.3.5

## Finding the Likelihood of an Event

Each time you roll a number cube, a number from 1 to 6 lands face up. This is called an *event*.

**Work with a partner to decide how many of the six possible results of rolling a number cube match the described event.**

**Then order the events from least likely (1) to most likely (9) by writing a number in each box to the right.**

Rolling a number less than 7 \_\_\_\_\_

Rolling an 8 \_\_\_\_\_

Rolling a number greater than 4 \_\_\_\_\_

Rolling a 5 \_\_\_\_\_

Rolling a number other than 6 \_\_\_\_\_

Rolling an even number \_\_\_\_\_

Rolling a number less than 5 \_\_\_\_\_

Rolling an odd number \_\_\_\_\_

Rolling a number divisible by 3 \_\_\_\_\_

### Reflect

1. Are any of the events impossible? \_\_\_\_\_

\_\_\_\_\_



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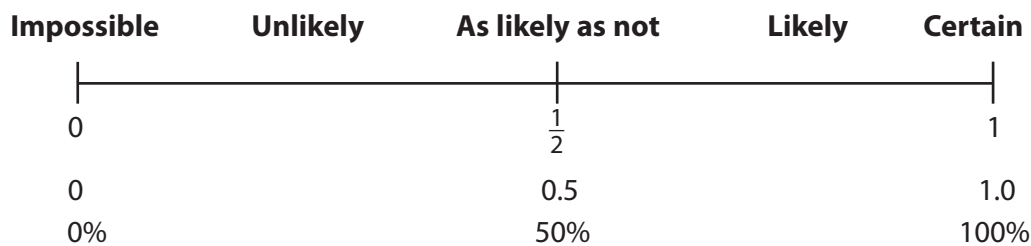
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# Describing Events

An **experiment** is an activity involving chance in which results are observed. Each observation of an experiment is a **trial**, and each result is an **outcome**. A set of one or more outcomes is an **event**.

The **probability** of an event, written  $P(\text{event})$ , measures the likelihood that the event will occur. Probability is a measure between 0 and 1 as shown on the number line, and can be written as a fraction, a decimal, or a percent.

If the event is not likely to occur, the probability of the event is close to 0. If an event is likely to occur, the event's probability is closer to 1.



## EXAMPLE 1



FL 7.SP.3.5

Tell whether each event is impossible, unlikely, as likely as not, likely, or certain. Then, tell whether the probability is 0, close to 0,  $\frac{1}{2}$ , close to 1, or 1.

- A** You roll a six-sided number cube and the number is 1 or greater.

This event is certain to happen. Its probability is 1.

Because you can roll the numbers 1, 2, 3, 4, 5, and 6 on a number cube, there are 6 possible outcomes.

- B** You roll two number cubes and the sum of the numbers is 3.

This event is unlikely to happen. Its probability is close to 0.

- C** A bowl contains disks marked with the numbers 1 through 10. You close your eyes and select a disk at random. You pick an odd number.

This event is as likely as not. The probability is  $\frac{1}{2}$ .

- D** A spinner has 8 equal sections marked 0 through 7. You spin and land on a prime number.

This event is as likely as not. The probability is  $\frac{1}{2}$ .

Remember that a prime number is a whole number greater than 1 and has exactly 2 divisors, 1 and itself.

### Math Talk

#### Mathematical Practices

Is an event that is *not* certain an impossible event? Explain.

### Reflect

2. The probability of event  $A$  is  $\frac{1}{3}$ . The probability of event  $B$  is  $\frac{1}{4}$ . What can you conclude about the two events?

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## YOUR TURN

3. A hat contains pieces of paper marked with the numbers 1 through 16. Tell whether picking an even number is impossible, unlikely, as likely as not, likely, or certain. Tell whether the probability is 0, close to 0,  $\frac{1}{2}$ , close to 1, or 1.

## Finding Probability

The **sample space** is a set of all possible outcomes for an event. A sample space can be small, such as the 2 outcomes when a coin is flipped. Or a sample space can be large, such as the possible number of Texas Classic automobile license plates. Identifying the sample space can help you calculate the probability of an event.



### Probability of An Event

$$P(\text{event}) = \frac{\text{number of times the event occurs}}{\text{total number of equally likely possible outcomes}}$$

## EXAMPLE 2



FL 7.SP.3.7a

What is the probability of rolling an even number on a standard number cube?

**STEP 1** Find the sample space for a standard number cube.

{1, 2, 3, 4, 5, 6} *There are 6 possible outcomes.*

**STEP 2** Find the number of ways to roll an even number.

2, 4, 6 *The event can occur 3 ways.*

**STEP 3** Find the probability of rolling an even number.

$$\begin{aligned} P(\text{even}) &= \frac{\text{number of ways to roll an even number}}{\text{number of faces on a number cube}} \\ &= \frac{3}{6} = \frac{1}{2} \quad \textit{Substitute values and simplify.} \end{aligned}$$

The probability of rolling an even number is  $\frac{1}{2}$ .



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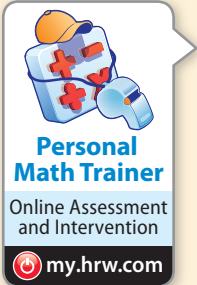
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## YOUR TURN

Find each probability. Write your answer in simplest form.

4. Picking a purple marble from a jar with 10 green and 10 purple marbles. \_\_\_\_\_
5. Rolling a number greater than 4 on a standard number cube. \_\_\_\_\_



## Using the Complement of an Event

The **complement** of an event is the set of all outcomes in the sample space that are *not* included in the event. For example, in the event of rolling a 3 on a number cube, the complement is rolling any number other than 3, which means the complement is rolling a 1, 2, 4, 5, or 6.

### An Event and Its Complement

The sum of the probabilities of an event and its complement equals 1.

$$P(\text{event}) + P(\text{complement}) = 1$$

You can apply probabilities to situations involving random selection, such as drawing a card out of a shuffled deck or pulling a marble out of a closed bag.

### EXAMPLE 3



FL 7.SP.3.7a

There are 2 red jacks in a standard deck of 52 cards. What is the probability of not getting a red jack if you select one card at random?

$$P(\text{event}) + P(\text{complement}) = 1$$

$$P(\text{red jack}) + P(\text{not a red jack}) = 1$$

$$\frac{2}{52} + P(\text{not a red jack}) = 1$$

$$\frac{2}{52} + P(\text{not a red jack}) = \frac{52}{52}$$

$$\begin{array}{r} \frac{2}{52} \\ \hline \end{array}$$

$$P(\text{not a red jack}) = \frac{50}{52}$$

$$P(\text{not a red jack}) = \frac{25}{26}$$

The probability of getting a red jack is  $\frac{2}{52}$ .

Substitute  $\frac{2}{52}$  for  $P(\text{red jack})$ .

Subtract  $\frac{2}{52}$  from both sides.

Simplify.

The probability that you will not draw a red jack is  $\frac{25}{26}$ . It is likely that you will not select a red jack.

## Reflect

6. Why do the probability of an event and the probability of its complement add up to 1?

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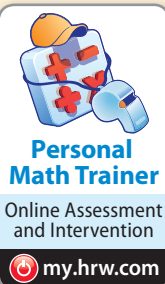
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## YOUR TURN

7. A jar contains 8 marbles marked with the numbers 1 through 8. You pick a marble at random. What is the probability of not picking the marble marked with the number 5? \_\_\_\_\_
8. You roll a standard number cube. Use the probability of rolling an even number to find the probability of rolling an odd number. \_\_\_\_\_



## Guided Practice

1. In a hat, you have index cards with the numbers 1 through 10 written on them. Order the events from least likely to happen (1) to most likely to happen (8) when you pick one card at random. In the boxes, write a number from 1 to 8 to order the eight different events. (*Explore Activity*)

You pick a number greater than 0.

You pick an even number.

You pick a number that is at least 2.

You pick a number that is at most 0.

You pick a number divisible by 3.

You pick a number divisible by 5.

You pick a prime number.

You pick a number less than the greatest prime number.

## Guided Practice

Determine whether each event is impossible, unlikely, as likely as not, likely, or certain. Then, tell whether the probability is 0, close to 0,  $\frac{1}{2}$ , close to 1, or 1. (Example 1)

2. randomly picking a green card from a standard deck of playing cards

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4. picking a number less than 15 from a jar with papers labeled from 1 to 12

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3. randomly picking a red card from a standard deck of playing cards

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5. picking a number that is divisible by 5 from a jar with papers labeled from 1 to 12

---

Find each probability. Write your answer in simplest form. (Example 2)

6. spinning a spinner that has 5 equal sections marked 1 through 5 and landing on an even number

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7. picking a diamond from a standard deck of playing cards which has 13 cards in each of four suits: spades, hearts, diamonds and clubs

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Use the complement to find each probability. (Example 3)

8. What is the probability of not rolling a 5 on a standard number cube?

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9. A spinner has 3 equal sections that are red, white, and blue. What is the probability of not landing on blue?

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10. A spinner has 5 equal sections marked 1 through 5. What is the probability of not landing on 4?

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11. There are 4 queens in a standard deck of 52 cards. You pick one card at random. What is the probability of not picking a queen?

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### ESSENTIAL QUESTION CHECK-IN

12. Describe an event that has a probability of 0% and an event that has a probability of 100%.

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# 12.1 Independent Practice



**FL** 7.SP.3.5, 7.SP.3.7a



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- 13.** There are 4 aces and 4 kings in a standard deck of 52 cards. You pick one card at random. What is the probability of selecting an ace or a king? Explain your reasoning.

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- 14.** There are 12 pieces of fruit in a bowl. Seven of the pieces are apples and two are peaches. What is the probability that a randomly selected piece of fruit will not be an apple or a peach? Justify your answer.

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- 15. Critique Reasoning** For breakfast, Clarissa can choose from oatmeal, cereal, French toast, or scrambled eggs. She thinks that if she selects a breakfast at random, it is likely that it will be oatmeal. Is she correct? Explain your reasoning.

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- 16. Draw Conclusions** A researcher’s garden contains 90 sweet pea plants, which have either white or purple flowers. About 70 of the plants have purple flowers, and about 20 have white flowers. Would you expect that one plant randomly selected from the garden will have purple or white flowers? Explain.




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- 17.** The power goes out as Sandra is trying to get dressed. If she has 4 white T-shirts and 10 colored T-shirts in her drawer, is it likely that she will pick a colored T-shirt in the dark? What is the probability she will pick a colored T-shirt? Explain your answers.

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**18.** James counts the hair colors of the 22 people in his class, including himself. He finds that there are 4 people with blonde hair, 8 people with brown hair, and 10 people with black hair. What is the probability that a randomly chosen student in the class does not have red hair? Explain.

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**19. Persevere in Problem Solving** A bag contains 8 blue coins and 6 red coins. A coin is removed at random and replaced by three of the other color.

**a.** What is the probability that the removed coin is blue?

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**b.** If the coin removed is blue, what is the probability of drawing a red coin after three red coins are put in the bag to replace the blue one?

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**c.** If the coin removed is red, what is the probability of drawing a red coin after three blue coins are put in the bag to replace the red one?

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**FOCUS ON HIGHER ORDER THINKING**

**20. Draw Conclusions** Give an example of an event in which all of the outcomes are not equally likely. Explain.

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**21. Critique Reasoning** A box contains 150 black pens and 50 red pens. Jose said the sum of the probability that a randomly selected pen will not be black and the probability that the pen will not be red is 1. Explain whether you agree.

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**22. Communicate Mathematical Ideas** A spinner has 7 identical sections. Two sections are blue, 1 is red, and 4 of the sections are green. Suppose the probability of an event happening is  $\frac{2}{7}$ . What does each number in the ratio represent? What outcome matches this probability?

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Work Area

# LESSON 12.2 Experimental Probability of Simple Events

 **FL** 7.SP.3.6

Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency ... Also 7.SP.3.7b



## ESSENTIAL QUESTION

How do you find the experimental probability of a simple event?

### EXPLORE ACTIVITY

 **FL** 7.SP.3.6, 7.SP.3.7b

## Finding Experimental Probability

You can toss a paper cup to demonstrate *experimental probability*.

- A** Consider tossing a paper cup. Fill in the Outcome column of the table with the three different ways the cup could land.
- B** Toss a paper cup twenty times. Record your observations in the table.

Outcome	Number of Times

### Reflect

- Do the outcomes appear to be equally likely? \_\_\_\_\_
- Describe the three outcomes using the words *likely* and *unlikely*.

\_\_\_\_\_

\_\_\_\_\_

- Use the number of times each event occurred to approximate the probability of each event.
- Make a Prediction** What do you think would happen if you performed more trials?

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

- What is the sum of the probabilities in 3?

\_\_\_\_\_

Outcome	Experimental Probability
Open-end up	$\frac{\text{open-end up}}{20} = \frac{\square}{20}$
Open-end down	$\frac{\text{open-end down}}{20} = \frac{\square}{20}$
On its side	$\frac{\text{on its side}}{20} = \frac{\square}{20}$



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# Calculating Experimental Probability

You can use *experimental probability* to approximate the probability of an event. An **experimental probability** of an event is found by comparing the number of times the event occurs to the total number of trials. When there is only one outcome for an event, it is called a **simple event**.

## Experimental Probability

For a given experiment:

$$\text{Experimental probability} = \frac{\text{number of times the event occurs}}{\text{total number of trials}}$$



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### EXAMPLE 1



FL 7.SP.3.7b

Martin has a bag of marbles. He removed one marble at random, recorded the color and then placed it back in the bag. He repeated this process several times and recorded his results in the table. Find the experimental probability of drawing each color.

Color	Frequency
Red	12
Blue	10
Green	15
Yellow	13

#### STEP 1

Identify the number of trials:  $12 + 10 + 15 + 13 = 50$

#### STEP 2

Complete the table of experimental probabilities. Write each answer as a fraction in simplest form.

Color	Experimental Probability
Red	$\frac{\text{frequency of the event}}{\text{total number of trials}} = \frac{12}{50} = \frac{6}{25}$
Blue	$\frac{\text{frequency of the event}}{\text{total number of trials}} = \frac{10}{50} = \frac{1}{5}$
Green	$\frac{\text{frequency of the event}}{\text{total number of trials}} = \frac{15}{50} = \frac{3}{10}$
Yellow	$\frac{\text{frequency of the event}}{\text{total number of trials}} = \frac{13}{50}$

Substitute the results recorded in the table. You can also write each probability as a decimal or as a percent.

### Reflect

6. **Communicate Mathematical Ideas** What are two different ways you could find the experimental probability of the event that Martin does not draw a red marble?

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## YOUR TURN

7. A spinner has three unequal sections: red, yellow, and blue. The table shows the results of Nolan's spins. Find the experimental probability of landing on each color. Write your answers in simplest form.

Color	Frequency
Red	10
Yellow	14
Blue	6

## Math Talk

### Mathematical Practices

Will everyone who does this experiment get the same results?

## Making Predictions with Experimental Probability

A **simulation** is a model of an experiment that would be difficult or inconvenient to actually perform. You can use a simulation to find an experimental probability and make a prediction.



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### EXAMPLE 2



FL 7.SP.3.6

A baseball team has a batting average of 0.250 so far this season. This means that the team's players get hits in 25% of their chances at bat. Use a simulation to predict the number of hits the team's players will have in their next 34 chances at bat.

- STEP 1** Choose a model.

$$\text{Batting average} = 0.250 = \frac{250}{1,000} = \frac{1}{4}$$

A standard deck of cards has four suits, hearts, diamonds, spades, and clubs. Since  $\frac{1}{4}$  of the cards are hearts, you can let hearts represent a "hit." Diamonds, clubs, and spades then represent "no hit."

- STEP 2** Perform the simulation.

Draw a card at random from the deck, record the result, and put the card back into the deck. Continue until you have drawn and replaced 34 cards in all.

(H = heart, D = diamond, C = club, S = spade)

H D D S H C H S D H C D C C D H H  
S D D H C C H C H H D S S S C H D

- STEP 3** Make a prediction.

Count the number of hearts in the simulation.

Since there are 11 hearts, you can predict that the team will have 11 hits in its next 34 chances at bat.

Since the team has 34 chances at bat, you must draw a card 34 times.

My Notes

## YOUR TURN

8. A toy machine has equal numbers of red, white, and blue foam balls which it releases at random. Ross wonders which color ball will be released next. Describe how you could use a standard number cube to predict the answer.

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## Guided Practice

1. A spinner has four sections lettered A, B, C, and D. The table shows the results of several spins. Find the experimental probability of spinning each letter as a fraction in simplest form, a decimal, and a percent.  
(Explore Activity and Example 1)

Letter	A	B	C	D
Frequency	14	7	11	8

A: \_\_\_\_\_ B: \_\_\_\_\_

C: \_\_\_\_\_ D: \_\_\_\_\_

2. Rachel's free-throw average for basketball is 60%. She wants to predict how many times in the next 50 tries she will make a free throw. Describe how she could use 10 index cards to predict the answer. (Example 2)

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### ESSENTIAL QUESTION CHECK-IN

3. **Essential Question Follow Up** How do you find an experimental probability of a simple event?

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# 12.2 Independent Practice



**FL** 7.SP.3.6, 7.SP.3.7b



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4. Dree rolls a strike in 6 out of the 10 frames of bowling. What is the experimental probability that Dree will roll a strike in the first frame of the next game? Explain why a number cube would not be a good way to simulate this situation.

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5. To play a game, you spin a spinner like the one shown. You win if the arrow lands in one of the areas marked "WIN". Lee played this game many times and recorded her results. She won 8 times and lost 40 times. Use Lee's data to explain how to find the experimental probability of winning this game.




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6. The names of the students in Mr. Hayes' math class are written on the board. Mr. Hayes writes each name on an index card and shuffles the cards. Each day he randomly draws a card, and the chosen student explains a math problem at the board. What is the probability that Ryan is chosen today? What is the probability that Ryan is **not** chosen today?

Anna	Alisha	Kenna	Bridget
Meghan	Cody	Parker	Grace
Michael	Gabe	Taylor	Joel
Kate	Kaylee	Shaw	Tessa
Jon	Ryan	Morgan	Leo

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7. **Critique Reasoning** A meteorologist reports an 80% chance of precipitation. Is this an example of experimental probability, written as a percent? Explain your reasoning.

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8. Mica and Joan are on the same softball team. Mica got 8 hits out of 48 times at bat, while Joan got 12 hits out of 40 times at bat. Who do you think is more likely to get a hit her next time at bat? Explain.

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9. **Make a Prediction** In tennis, Gabby serves an ace, a ball that can't be returned, 4 out of the 10 times she serves. What is the experimental probability that Gabby will serve an ace in the first match of the next game? Make a prediction about how many aces Gabby will have for the next 40 serves. Justify your reasoning.

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10. **Represent Real-World Problems** Patricia finds that the experimental probability that her dog will want to go outside between 4 P.M. and 5 P.M. is  $\frac{7}{12}$ . About what percent of the time does her dog **not** want to go out between 4 P.M. and 5 P.M.?

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**H.O.T.** FOCUS ON HIGHER ORDER THINKING

11. **Explain the Error** Talia tossed a penny many times. She got 40 heads and 60 tails. She said the experimental probability of getting heads was  $\frac{40}{60}$ . Explain and correct her error.

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12. **Communicate Mathematical Ideas** A high school has 438 students, with about the same number of males as females. Describe a simulation to predict how many of the first 50 students who leave school at the end of the day are female.

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13. **Critical Thinking** For a scavenger hunt, Chessa put one coin in each of 10 small boxes. Four coins are quarters, 4 are dimes, and 2 are nickels. How could you simulate choosing one box at random? Would you use the same simulation if you planned to put these coins in your pocket and choose one? Explain your reasoning.

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Work Area

# LESSON 12.3 Experimental Probability of Compound Events

 **FL** 7.SP.3.8

Find probabilities of compound events using... tables, ... , and simulation. Also 7.SP.3.8a, 7.SP.3.8b, 7.SP.3.8c



## ESSENTIAL QUESTION

How do you find the experimental probability of a compound event?

### EXPLORE ACTIVITY

 **FL** 7.SP.3.8a, 7.SP.3.8b

## Exploring Compound Probability

A **compound event** is an event that includes two or more simple events, such as flipping a coin *and* rolling a number cube. A compound event can include events that depend on each other or are independent. Events are independent if the occurrence of one event does not affect the probability of the other event, such as flipping a coin and rolling a number cube.

- A** What are the possible outcomes of flipping a coin once? \_\_\_\_\_
- B** What are the possible outcomes of rolling a standard number cube once? \_\_\_\_\_
- C** Complete the list for all possible outcomes for flipping a coin *and* rolling a number cube.

H1, H2, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, T1, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_

There are \_\_\_\_\_ possible outcomes for this compound event.

- D** Flip a coin and roll a number cube 50 times. Use tally marks to record your results in the table.

	1	2	3	4	5	6
H						
T						

- E** Based on your data, which compound event had the greatest experimental probability and what was it? The least experimental probability? \_\_\_\_\_

- F** **Draw Conclusions** Did you expect to have the same probability for each possible combination of flips and rolls? Why or why not?

\_\_\_\_\_

\_\_\_\_\_

H1 would mean the coin landed on heads, and the number cube showed a 1.



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# Calculating Experimental Probability of Compound Events

The experimental probability of a compound event can be found using recorded data.

## EXAMPLE 1



FL 7.SP.3.8, 7.SP.3.8a

A food trailer serves chicken and records the order size and sides on their orders, as show in the table. What is the experimental probability that the next order is for 3-pieces with cole slaw?

	Green Salad	Macaroni & Cheese	French Fries	Cole Slaw
2 pieces	33	22	52	35
3 pieces	13	55	65	55

### STEP 1

Find the total number of trials, or orders.

$$33 + 22 + 52 + 35 + 13 + 55 + 65 + 55 = 330$$

### STEP 2

Find the number of orders that are for 3 pieces with cole slaw: 55.

### STEP 3

Find the experimental probability.

$$\begin{aligned}
 P(3 \text{ piece} + \text{slaw}) &= \frac{\text{number of 3 piece} + \text{slaw}}{\text{total number of orders}} \\
 &= \frac{55}{330} \quad \text{Substitute the values.} \\
 &= \frac{1}{6} \quad \text{Simplify.}
 \end{aligned}$$

The experimental probability that the next order is for 3 pieces of chicken with cole slaw is  $\frac{1}{6}$ .

## Math Talk

### Mathematical Practices

Javier said the total number of orders is 8 and not 330. Is he correct? Explain.

## YOUR TURN

1. Drink sales for an afternoon at the school carnival were recorded in the table. What is the experimental probability that the next drink is a small cocoa?

	Soda	Water	Cocoa
Small	77	98	60
Large	68	45	52

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# Using a Simulation to Make a Prediction

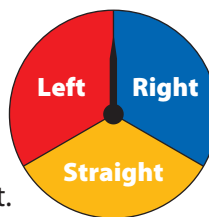
You can use a simulation or model of an experiment to find the experimental probability of compound events.



**EXAMPLE 2**   **FL** 7.SP.3.8c

At a street intersection, a vehicle is classified either as a *car* or a *truck*, and it can turn *left*, *right*, or go *straight*. About an equal number of cars and trucks go through the intersection and turn in each direction. Use a simulation to find the experimental probability that the next vehicle will be a car that turns right.

- STEP 1** Choose a model.  
Use a coin toss to model the two vehicle types.  
Let Heads = **C**ar and Tails = **T**ruck
- Use a spinner divided into 3 equal sectors to represent the *three* directions as shown.



- STEP 2** Find the sample space for the compound event.  
There are 6 possible outcomes: **CL, CR, CS, TL, TR, TS**

- STEP 3** Perform the simulation.  
A coin was tossed and a spinner spun 50 times.  
The results are shown in the table.

	Car	Truck
Left	8	9
Right	6	11
Straight	9	7

- STEP 4** Find the experimental probability that a car turns right.

$$\begin{aligned}
 P(\text{Car turns right}) &= \frac{\text{frequency of compound event}}{\text{total number of trials}} \\
 &= \frac{6}{50} \quad \text{Substitute the values.} \\
 &= \frac{3}{25} \quad \text{Simplify.}
 \end{aligned}$$

Based on the simulation, the experimental probability is  $\frac{3}{25}$  that the next vehicle will be a car that turns right.

## Reflect

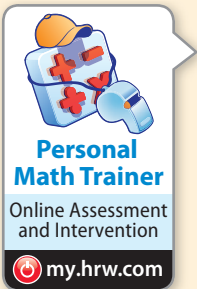
2. **Make a Prediction** Predict the number of cars that turn right out of 100 vehicles that enter the intersection. Explain your reasoning.

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My Notes



## YOUR TURN

3. A jeweler sells necklaces made in three sizes and two different metals. Use the data from a simulation to find the experimental probability that the next necklace sold is a 20-inch gold necklace.

	Silver	Gold
12 in.	12	22
16 in.	16	8
20 in.	5	12

\_\_\_\_\_

## Guided Practice

1. A dentist has 400 male and female patients that range in ages from 10 years old to 50 years old and up as shown in the table. What is the experimental probability that the next patient will be female and in the age range 22–39? (*Explore Activity and Example 1*)

	Range: 10–21	Range: 22–39	Range: 40–50	Range: 50+
Male	44	66	32	53
Female	36	50	45	74

\_\_\_\_\_

2. At a car wash, customers can choose the type of wash and whether to use the interior vacuum. Customers are equally likely to choose each type of wash and whether to use the vacuum. Use a simulation to find the experimental probability that the next customer purchases a deluxe wash and no interior vacuum. Describe your simulation. (*Example 2*)



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\_\_\_\_\_  
\_\_\_\_\_

## ESSENTIAL QUESTION CHECK-IN

3. How do you find the experimental probability of a compound event?


\_\_\_\_\_  
\_\_\_\_\_



# 12.3 Independent Practice



**FL** 7.SP.3.8, 7.SP.3.8a, 7.SP.3.8b, 7.SP.3.8c



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4. **Represent Real-World Problems** For the same food trailer mentioned in Example 1, explain how to find the experimental probability that the next order is two pieces of chicken with a green salad.

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The school store sells spiral notebooks in four colors and three different sizes. The table shows the sales by size and color for 400 notebooks.

	Red	Green	Blue	Yellow
<b>100 Pages</b>	55	37	26	12
<b>150 Pages</b>	60	44	57	27
<b>200 Pages</b>	23	19	21	19

5. What is the experimental probability that the next customer buys a red notebook with 150 pages?
6. What is the experimental probability that the next customer buys any red notebooks?
7. **Analyze Relationships** How many possible combined page count and color choices are possible? How does this number relate to the number of page size choices and to the number of color choices?

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A middle school English teacher polled random students about how many pages of a book they read per week.

	6th	7th	8th
<b>75 Pages</b>	24	18	22
<b>100 Pages</b>	22	32	24
<b>150 Pages</b>	30	53	25

8. **Critique Reasoning** Jennie says the experimental probability that a 7th grade student reads at least 100 pages per week is  $\frac{16}{125}$ . What is her error and the correct experimental probability?
9. **Analyze Relationships** Based on the data, which group(s) of students should be encouraged to read more? Explain your reasoning.

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10. **Make a Conjecture** Would you expect the probability for the simple event “rolling a 6” to be greater than or less than the probability of the compound event “rolling a 6 and getting heads on a coin”? Explain.

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11. **Critique Reasoning** Donald says he uses a standard number cube for simulations that involve 2, 3, or 6 equal outcomes. Explain how Donald can do this.

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12. **Draw Conclusions** Data collected in a mall recorded the shoe styles worn by 150 male and for 150 female customers. What is the probability that the next customer is male and has an open-toe shoe (such as a sandal)? What is the probability that the next male customer has an open-toe shoe? Are the two probabilities the same? Explain.

	Male	Female
Open toe	11	92
Closed toe	139	58

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13. **What If?** Suppose you wanted to perform a simulation to model the shoe style data shown in the table. Could you use two coins? Explain.

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14. **Represent Real-World Problems** A middle school is made up of grades 6, 7, and 8, and has about the same number of male and female students in each grade. Explain how to use a simulation to find the experimental probability that the first 50 students who arrive at school are male and 7th graders.

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# LESSON 12.4 Making Predictions with Experimental Probability

 **FL** 7.SP.3.6

Approximate the probability of a chance event by collecting data on the chance process that produces it. . . , and predict the approximate relative frequency given the probability.



## ESSENTIAL QUESTION

How do you make predictions using experimental probability?

## Using Experimental Probability to Make a Prediction

Scientists study data to make predictions. You can use probabilities to make predictions in your daily life.



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### EXAMPLE 1



**FL** 7.SP.3.6

Danae found that the experimental probability of her making a bull's-eye when throwing darts is  $\frac{2}{10}$ , or 20%. Out of 75 throws, about how many bull's-eyes could she predict she would make?

**Method 1: Use a proportion.**

$$\frac{2}{10} = \frac{x}{75}$$

Write a proportion. 2 out of 10 is how many out of 75?

$$\frac{2}{10} = \frac{x}{75}$$

$$\frac{2}{10} = \frac{15}{75}$$

$$x = 15$$

Since 10 times 7.5 is 75, multiply 2 times 7.5 to find the value of  $x$ .



**Method 2: Use a percent equation.**

$$0.20 \cdot 75 = x$$

$$15 = x$$

You can write probabilities as ratios, decimals, or percents.

Danae can predict that she will make about 15 bull's-eye throws out of 75.

### YOUR TURN

1. A car rental company sells accident insurance to 24% of its customers. Out of 550 customers, about how many customers are predicted to purchase insurance? \_\_\_\_\_



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# Using Experimental Probability to Make a Qualitative Prediction

A prediction is something you reasonably expect to happen in the future. A qualitative prediction helps you decide which situation is more likely in general.

## EXAMPLE 2



FL 7.SP.3.6

A doctor's office records data and concludes that, on average, 11% of patients call to reschedule their appointments per week. The office manager predicts that 23 appointments will be rescheduled out of the 240 total appointments during next week. Explain whether the prediction is reasonable.

**Method 1: Use a proportion.**

$$\frac{11}{100} = \frac{x}{240}$$

Write a proportion. 11 out of 100 is how many out of 240?

$$\frac{11}{100} = \frac{x}{240}$$

$$\times 2.4$$

$$\frac{11}{100} = \frac{26.4}{240}$$

$$\times 2.4$$

$$x = 26.4$$

Since 100 times 2.4 is 240, multiply 11 times 2.4 to find the value of x.

26.4 is the average number of patients that would call to reschedule.

**Method 2: Use a percent equation.**

$$0.11 \cdot 240 = x$$

Find 11% of 240.

$$26.4 = x$$

Solve for x.

The prediction of 23 is reasonable but a little low, because 23 is a little less than 26.4.

## Reflect

2. Does 26.4 make sense for the number of patients?

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## YOUR TURN

3. In emails to monthly readers of a newsletter 3% of the emails come back undelivered. The editor predicts that if he sends out 12,372 emails, he will receive 437 notices for undelivered email. Do you agree with his prediction?

Explain. \_\_\_\_\_  
\_\_\_\_\_



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# Making a Quantitative Prediction

You can use proportional reasoning to make quantitative predictions and compare options in real-world situations.



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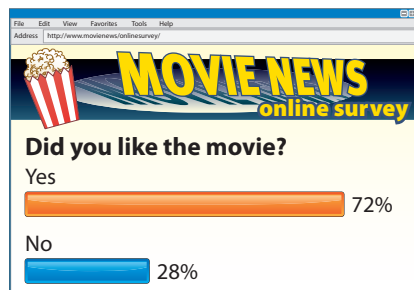
## EXAMPLE 3

Problem Solving



FL 7.SP.3.6

An online poll for a movie site shows its polling results for a new movie. If a newspaper surveys 150 people leaving the movie, how many people can it predict will like the movie based on the online poll? Is the movie site's claim accurate if the newspaper has 104 people say they like the movie?



### Analyze Information

The **answer** is a prediction for how many people out of 150 will like the movie based on the online poll. Also tell whether the 104 people that say they like the movie is enough to support the movie site's claim.

#### List the important information:

- The online poll says 72% of movie goers like the new movie.
- A newspaper surveys 150 people.



### Formulate a Plan

Use a proportion to calculate 72% of the 150 people surveyed.



### Solve

$$\frac{72}{100} = \frac{x}{150}$$

Set up a proportion. 72 out of 100 is how many out of 150?

$$\frac{72}{100} = \frac{x}{150}$$

$$\frac{72}{100} \times 1.5 = \frac{x}{150} \times 1.5$$

$$\frac{72}{100} = \frac{108}{150}$$

$$\frac{72}{100} \times 1.5 = \frac{108}{150} \times 1.5$$

$$x = 108$$

Since 100 times 1.5 is 150, multiply 72 times 1.5 to find the value of  $x$ .

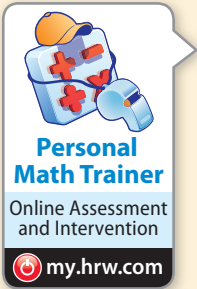
The newspaper can predict that 108 out of 150 people will say they like the movie, based on the online poll.



### Justify and Evaluate

Since 108 is close to 104, the newspaper survey and the online poll show that about the same percent of people like the movie.

My Notes



## YOUR TURN

4. On average, 24% of customers who buy shoes in a particular store buy two or more pairs. One weekend, 350 customers purchased shoes. How many can be predicted to buy two or more pairs? If 107 customers buy more than two pairs, did more customers than normal buy two or more pairs?

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## Guided Practice

1. A baseball player reaches first base 30% of the times he is at bat. Out of 50 times at bat, about how many times will the player reach first base? (Example 1)  

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2. The experimental probability that it will rain on any given day in Houston, Texas, is about 15%. Out of 365 days, about how many days can residents predict rain? (Example 1)  

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3. A catalog store has 6% of its orders returned for a refund. The owner predicts that a new candle will have 812 returns out of the 16,824 sold. Do you agree with this prediction? Explain. (Example 2)  

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4. On a toy assembly line, 3% of the toys are found to be defective. The quality control officer predicts that 872 toys will be found defective out of 24,850 toys made. Do you agree with this prediction? Explain. (Example 2)  

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5. A light-rail service claims to be on time 98% of the time. Jeanette takes the light-rail 40 times one month, how many times can she predict she will be on time? Is the light-rail's claim accurate if she is late 6 times? (Example 3)  

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6. On average, a college claims to accept 18% of its applicants. If the college has 5,000 applicants, predict how many will be accepted. If 885 applicants are accepted, is the college's claim accurate? (Example 3)  

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## ESSENTIAL QUESTION CHECK-IN

7. How do you make predictions using experimental probability?


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# 12.4 Independent Practice

 **FL** 7.SP.3.6



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The table shows the number of students in a middle school at the beginning of the year and the percentage that can be expected to move out of the area by the end of the year.

	6th	7th	8th
<b>Number of Students</b>	250	200	150
<b>% Moves</b>	2%	4%	8%

- 8.** How many 7th grade students are expected to move by the end of the year? If 12 students actually moved, did more or fewer 7th grade students move than expected? Justify your answer.

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- 9. Critique Reasoning** The middle school will lose some of its funding if 50 or more students move away in any year. The principal claims he only loses about 30 students a year. Do the values in the table support his claim? Explain.

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- 10. Represent Real-World Problems** An airline knows that, on average, the probability that a passenger will not show up for a flight is 6%. If an airplane is fully booked and holds 300 passengers, how many seats are expected to be empty? If the airline overbooked the flight by 10 passengers, about how many passengers are expected to show up for the flight? Justify your answer.

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- 11. Draw Conclusions** In a doctor’s office, an average of 94% of the clients pay on the day of the appointment. If the office has 600 clients per month, how many are expected not to pay on the day of the appointment? If 40 clients do not pay on the day of their appointment in a month, did more or fewer than the average not pay?

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- 12. Counterexamples** The soccer coach claimed that, on average, only 80% of the team come to practice each day. The table shows the number of students that came to practice for 8 days. If the team has 20 members, how many team members should come to practice to uphold the coach's claim? Was the coach's claim accurate? Explain your reasoning.

	1	2	3	4	5	6	7	8
Number of Students	18	15	18	17	17	19	20	20

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- 13. What's the Error?** Ronnie misses the school bus 1 out of every 30 school days. He sets up the proportion  $\frac{1}{30} = \frac{180}{x}$  to predict how many days he will miss the bus in the 180-day school year. What is Ronnie's error?

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**FOCUS ON HIGHER ORDER THINKING**

- 14. Persevere in Problem Solving** A gas pump machine rejects 12% of credit card transactions. If this is twice the normal rejection rate for a normal gas pump, how many out of 500 credit cards transactions would a normal gas pump machine reject? \_\_\_\_\_
- 15. Make Predictions** An airline's weekly flight data showed a 98% probability of being on time. If this airline has 15,000 flights in a year, how many flights would you predict to arrive on time? Explain whether you can use the data to predict whether a specific flight with this airline will be on time.

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- 16. Draw Conclusions** An average response rate for a marketing letter is 4%, meaning that 4% of the people who receive the letter respond to it. A company writes a new type of marketing letter, sends out 2,400 of them, and gets 65 responses. Explain whether the new type of letter would be considered to be a success.

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Work Area



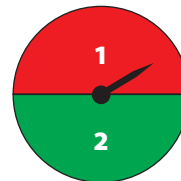
# Ready to Go On?



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## 12.1 Probability

- Josue tosses a coin and spins the spinner at the right. What are all the possible outcomes?



\_\_\_\_\_

## 12.2 Experimental Probability of Simple Events

- While bowling with friends, Brandy rolls a strike in 6 out of 10 frames. What is the experimental probability that Brandy will roll a strike in the first frame of the next game?
- Ben is greeting customers at a music store. Of the first 20 people he sees enter the store, 13 are wearing jackets and 7 are not. What is the experimental probability that the next person to enter the store will be wearing a jacket?

\_\_\_\_\_

\_\_\_\_\_

## 12.3 Experimental Probability of Compound Events

- Auden rolled two number cubes and recorded the results.

Roll #1	Roll #2	Roll #3	Roll #4	Roll #5	Roll #6	Roll #7
2, 1	4, 5	3, 2	2, 2	1, 3	6, 2	5, 3

What is the experimental probability that the sum of the next two numbers rolled is greater than 5?

\_\_\_\_\_

## 12.4 Making Predictions with Experimental Probability

- A player on a school baseball team reaches first base  $\frac{3}{10}$  of the time he is at bat. Out of 80 times at bat, about how many times would you predict he will reach first base?

\_\_\_\_\_

### ESSENTIAL QUESTION

- How is experimental probability used to make predictions?

\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_



# Assessment Readiness



## Selected Response

1. A frozen yogurt shop offers scoops in cake cones, waffle cones, or cups. You can get vanilla, chocolate, strawberry, pistachio, or coffee flavored frozen yogurt. If you order a single scoop, how many outcomes are in the sample space?

- (A) 3                      (C) 8  
(B) 5                      (D) 15

2. A bag contains 7 purple beads, 4 blue beads, and 4 pink beads. What is the probability of **not** drawing a pink bead?

- (A)  $\frac{4}{15}$                       (C)  $\frac{8}{15}$   
(B)  $\frac{7}{15}$                       (D)  $\frac{11}{15}$

3. During the month of June, Ava kept track of the number of days she saw birds in her garden. She saw birds on 18 days of the month. What is the experimental probability that she will see birds in her garden on July 1?

- (A)  $\frac{1}{18}$                       (C)  $\frac{1}{2}$   
(B)  $\frac{2}{5}$                       (D)  $\frac{3}{5}$

4. A rectangle has a width of 4 inches and a length of 6 inches. A similar rectangle has a width of 12 inches. What is the length of the similar rectangle?

- (A) 8 inches                      (C) 14 inches  
(B) 12 inches                      (D) 18 inches

5. The experimental probability of hearing thunder on any given day in Ohio is 30%. Out of 600 days, on about how many days can Ohioans expect to hear thunder?

- (A) 90 days                      (C) 210 days  
(B) 180 days                      (D) 420 days

6. Isidro tossed two coins several times and then recorded the results in the table below.

Toss 1	Toss 2	Toss 3	Toss 4	Toss 5
H;T	T;T	T;H	H;T	H;H

What is the experimental probability that both coins will land on the same side on Isidro's next toss?

- (A)  $\frac{1}{5}$                       (C)  $\frac{3}{5}$   
(B)  $\frac{2}{5}$                       (D)  $\frac{4}{5}$

## Mini-Task

7. Magdalena had a spinner that was evenly divided into sections of red, blue, and green. She spun the spinner and tossed a coin several times. The table below shows the results.

Trial 1	Trial 2	Trial 3	Trial 4	Trial 5
blue; T	green; T	green; H	red; T	blue; H

a. What are all the possible outcomes?

---



---

b. What experimental probability did Magdalena find for spinning blue? Give your answer as a fraction in simplest form, as a decimal, and as a percent.

---

c. Out of 90 trials, how many times should Magdalena predict she will spin green while tossing tails?

---

# Theoretical Probability and Simulations

MODULE



# 13



## ESSENTIAL QUESTION

How can you use theoretical probability to solve real-world problems?



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### Real-World Video

Many carnival games rely on theoretical probability to set the chance of winning fairly low. Understanding how the game is set up might help you be more likely to win.

LESSON 13.1

### Theoretical Probability of Simple Events



FL 7.SP.3.7, 7.SP.3.7a

LESSON 13.2

### Theoretical Probability of Compound Events



FL 7.SP.3.8, 7.SP.3.8a, 7.SP.3.8b

LESSON 13.3

### Making Predictions with Theoretical Probability



FL 7.SP.3.6, 7.SP.3.7a

LESSON 13.4

### Using Technology to Conduct a Simulation



FL 7.SP.3.8, 7.SP.3.8c

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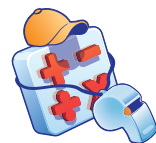
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# Are YOU Ready?

Complete these exercises to review skills you will need for this module.



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## Fractions, Decimals, and Percents

**EXAMPLE** Write  $\frac{3}{8}$  as a decimal and a percent.

$$\begin{array}{r} 0.375 \\ 8 \overline{)3.000} \\ \underline{-24} \phantom{00} \\ 60 \\ \underline{-56} \\ 40 \\ \underline{-40} \\ 0 \end{array}$$

Write the fraction as a division problem. Write a decimal point and zeros in the dividend.

Place a decimal point in the quotient. Divide as with whole numbers.

$$0.375 = 37.5\%$$

Write the decimal as a percent.

Write each fraction as a decimal and a percent.

1.  $\frac{3}{4}$  \_\_\_\_\_

2.  $\frac{2}{5}$  \_\_\_\_\_

3.  $\frac{9}{10}$  \_\_\_\_\_

4.  $\frac{7}{20}$  \_\_\_\_\_

5.  $\frac{7}{8}$  \_\_\_\_\_

6.  $\frac{1}{20}$  \_\_\_\_\_

7.  $\frac{19}{25}$  \_\_\_\_\_

8.  $\frac{23}{50}$  \_\_\_\_\_

## Operations with Fractions

**EXAMPLE**  $1 - \frac{7}{12} = \frac{12}{12} - \frac{7}{12}$   
 $= \frac{12-7}{12}$   
 $= \frac{5}{12}$

Use the denominator of the fraction to write 1 as a fraction. Subtract the numerators.

Simplify.

Find each difference.

9.  $1 - \frac{1}{5}$  \_\_\_\_\_

10.  $1 - \frac{2}{9}$  \_\_\_\_\_

11.  $1 - \frac{8}{13}$  \_\_\_\_\_

12.  $1 - \frac{3}{20}$  \_\_\_\_\_

## Multiply Fractions

**EXAMPLE**  $\frac{4}{15} \times \frac{5}{6} = \frac{\cancel{4}^2}{15} \times \frac{\cancel{5}_3}{6}$   
 $= \frac{2}{9}$

Divide by the common factors.

Simplify.

Multiply. Write each product in simplest form.

13.  $\frac{8}{15} \times \frac{5}{8}$  \_\_\_\_\_

14.  $\frac{2}{9} \times \frac{3}{4}$  \_\_\_\_\_

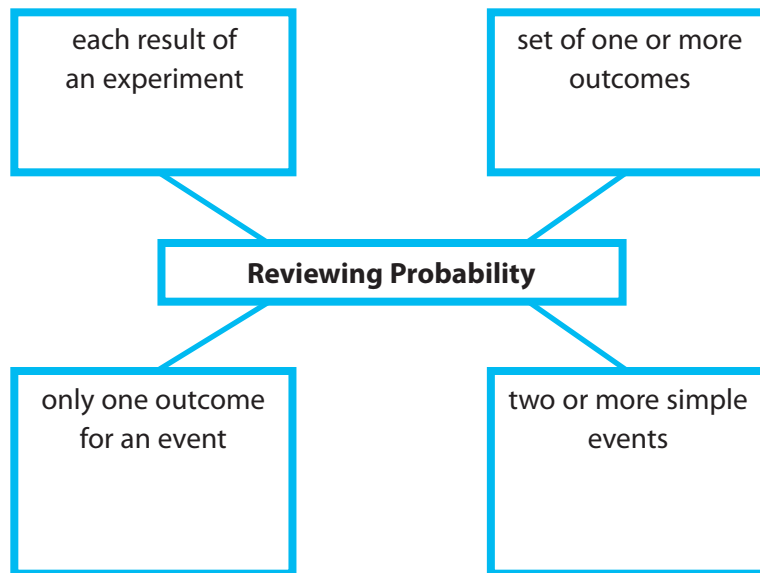
15.  $\frac{9}{16} \times \frac{12}{13}$  \_\_\_\_\_

16.  $\frac{7}{10} \times \frac{5}{28}$  \_\_\_\_\_

# Reading Start-Up

## Visualize Vocabulary

Use the ✓ words to complete the graphic.



## Vocabulary

### Review Words

- complement  
(*complemento*)
- ✓ compound event (*suceso compuesto*)
- ✓ event (*suceso*)  
experiment (*experimento*)
- ✓ outcome (*resultado*)
- ✓ simple event (*suceso simple*)  
probability (*probabilidad*)

### Preview Words

- theoretical probability  
(*probabilidad teórica*)

## Understand Vocabulary

Match the term on the left to the correct expression on the right.

- |                            |   |
|----------------------------|---|
| 1. compound event          | A. The set of all outcomes that are not the desired event.  |
| 2. theoretical probability | B. An event made of two or more simple events.  |
| 3. complement              | C. The ratio of the number of equally likely outcomes in an event to the total number of possible outcomes. |

## Active Reading

**Two-Panel Flip Chart** Create a two-panel flip chart, to help you understand the concepts in this module. Label one flap “Simple Events” and the other flap “Compound Events.” As you study each lesson, write important ideas under the appropriate flap. Include information that will help you remember the concepts later when you look back at your notes.





# Unpacking the Standards

Understanding the standards and the vocabulary terms in the standards will help you know exactly what you are expected to learn in this module.

**FL 7.SP.3.7a**

Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events.

## What It Means to You

You will find the probabilities of a simple event and its complement.

### UNPACKING EXAMPLE 7.SP.3.7A

Tara has a bag that contains 8 white marbles, 10 green marbles, and 7 red marbles. She selects a marble at random. Find the probability that the marble is red, and the probability that it is **not** red.

$$P(\text{red}) = \frac{\text{number of red marbles}}{\text{total number of marbles}}$$

$$= \frac{7}{25}$$

$$P(\text{not red}) = 1 - P(\text{red}) = 1 - \frac{7}{25} = \frac{25}{25} - \frac{7}{25} = \frac{18}{25}$$

The probability that the marble is red is  $\frac{7}{25}$ , and the probability that it is not red is  $\frac{18}{25}$ .

**FL 7.SP.3.8b**

Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event.

### Key Vocabulary

**compound event** (*suceso compuesto*)

An event made of two or more simple events.

## What It Means to You

You will identify the outcomes in the sample space of a compound event.

### UNPACKING EXAMPLE 7.SP.3.8B

Identify the sample space for flipping a coin and rolling a number cube.

Make a table to organize the information.

		Number Cube Outcomes					
		1	2	3	4	5	6
C O I N	H	H1	H2	H3	H4	H5	H6
	T	T1	T2	T3	T4	T5	T6

The sample space includes 12 possible outcomes: H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, and T6.



Visit [my.hrw.com](http://my.hrw.com) to see all **Florida Math Standards** unpacked.

# LESSON 13.1 Theoretical Probability of Simple Events

 **FL** 7.SP.3.7a

Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. Also 7.SP.3.6, 7.SP.3.7



## ESSENTIAL QUESTION

How can you find the theoretical probability of a simple event?

### EXPLORE ACTIVITY 1



 **FL** 7.SP.3.7a

## Finding Theoretical Probability

In previous lessons, you found probabilities based on observing data, or experimental probabilities. In this lesson, you will find *theoretical probabilities*.

**At a school fair, you have a choice of spinning Spinner A or Spinner B. You win an MP3 player if the spinner lands on a section with a star in it. Which spinner should you choose if you want a better chance of winning?**

**A** Complete the table.

	Spinner A	Spinner B
<b>Total number of outcomes</b>		
<b>Number of sections with stars</b>		
<b><math>P(\text{winning MP3})</math></b> = $\frac{\text{number of sections with stars}}{\text{total number of outcomes}}$		

**B** Compare the ratios for Spinner A and Spinner B.

The ratio for Spinner \_\_\_\_\_ is greater than the ratio for Spinner \_\_\_\_\_.

I should choose \_\_\_\_\_ for a better chance of winning.

### Reflect

- Theoretical probability* is a way to describe how you found the chance of winning an MP3 player in the scenario above. Using the spinner example to help you, explain in your own words how to find the theoretical probability of an event.

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Spinner A



Spinner B



### Math Talk

#### Mathematical Practices



Describe a way to change Spinner B to make your chances of winning equal to your chances of not winning. Explain.



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# Calculating Theoretical Probability of Simple Events

**Theoretical probability** is the probability that an event occurs when all of the outcomes of the experiment are equally likely.

## Theoretical Probability

$$P(\text{event}) = \frac{\text{number of ways the event can occur}}{\text{total number of equally likely outcomes}}$$

Probability can be written as a fraction, a decimal, or a percent. For example, the probability you win with Spinner B is  $\frac{5}{16}$ . You can also write that as 0.3125 or as 31.25%.

### EXAMPLE 1



FL 7.SP.3.7a

A bag contains 6 red marbles and 12 blue ones. You select one marble at random from the bag. What is the probability that you select a red marble? Write your answer in simplest form.

**STEP 1** Find the number of ways the event can occur, that is, the number of red marbles: 6

**STEP 2** Add to find the total number of equally likely outcomes.

$$\begin{array}{rcccl} \text{number of red} & + & \text{number of blue} & = & \text{total number} \\ \text{marbles} & & \text{marbles} & & \text{of marbles} \\ 6 & + & 12 & = & 18 \end{array}$$

There are 18 possible outcomes in the sample space.

**STEP 3** Find the probability of selecting a red marble.

$$P(\text{red marble}) = \frac{\text{number of red marbles}}{\text{total number of marbles}} = \frac{6}{18}$$

The probability that you select a red marble is  $\frac{6}{18}$ , or  $\frac{1}{3}$ .

### Math Talk

#### Mathematical Practices

Describe a situation that has a theoretical probability of  $\frac{1}{4}$ .

### YOUR TURN

2. You roll a number cube one time. What is the probability that you roll a 3 or 4? Write your answer in simplest form.

$$P(\text{rolling a 3 or 4}) = \frac{\boxed{\phantom{0000}}}{\boxed{\phantom{0000}}} = \frac{\boxed{\phantom{00}}}{\boxed{\phantom{00}}} = \frac{\boxed{\phantom{00}}}{\boxed{\phantom{00}}}$$

3. How is the sample space for an event related to the formula for theoretical probability? \_\_\_\_\_

\_\_\_\_\_



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# Comparing Theoretical and Experimental Probability

Now that you have calculated theoretical probabilities, you may wonder how theoretical and experimental probabilities compare.

Six students are performing in a talent contest. You roll a number cube to determine the order of the performances.

**STEP 1** You roll the number cube once. Complete the table of theoretical probabilities for the different outcomes.

Number	1	2	3	4	5	6
Theoretical probability						

**STEP 2** Predict the number of times each number will be rolled out of 30 total rolls.

1:  times      3:  times      5:  times  
 2:  times      4:  times      6:  times

**STEP 3** Roll a number cube 30 times. Complete the table for the frequency of each number and then find its experimental probability.

Number	1	2	3	4	5	6
Frequency						
Experimental probability						

**STEP 4** Look at the tables you completed. How do the experimental probabilities compare with the theoretical probabilities?

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**STEP 5** **Conjecture** By performing more trials, you tend to get experimental results that are closer to the theoretical probabilities. Combine your table from **Step 3** with those of your classmates to make one table for the class. How do the class experimental probabilities compare with the theoretical probabilities?

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**EXPLORE ACTIVITY 2** (cont'd)

**Reflect**

4. Could the experimental probabilities ever be exactly equal to the theoretical probability? If so, how likely is it? If not, why not?

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**Guided Practice**

At a school fair, you have a choice of randomly picking a ball from Basket A or Basket B. Basket A has 5 green balls, 3 red balls, and 8 yellow balls. Basket B has 7 green balls, 4 red balls, and 9 yellow balls. You can win a digital book reader if you pick a red ball. (Explore Activity 1)

	Basket A	Basket B
Total number of outcomes		
Number of red balls		
$P(\text{win}) =$ $\frac{\text{number of red balls}}{\text{total number of outcomes}}$		

- Complete the chart. Write each answer in simplest form.
- Which basket should you choose if you want the better chance of winning? \_\_\_\_\_

A spinner has 11 equal-sized sections marked 1 through 11. Find each probability. (Example 1)

- You spin once and land on an odd number.  
 $P(\text{odd}) = \frac{\text{number of sections}}{\text{total number of}} = \frac{\boxed{\phantom{00}}}{\boxed{\phantom{00}}}$
- You spin once and land on an even number.  
 $P(\text{even}) = \frac{\text{number of sections}}{\text{total number of}} = \frac{\boxed{\phantom{00}}}{\boxed{\phantom{00}}}$

You roll a number cube once.

- What is the theoretical probability that you roll a 3 or 4? (Example 1) \_\_\_\_\_
- Suppose you rolled the number cube 199 more times. Would you expect the experimental probability of rolling a 3 or 4 to be the same as your answer to Exercise 5? (Explore Activity 2)

---

**ESSENTIAL QUESTION CHECK-IN**


- How can you find the probability of a simple event if the total number of equally likely outcomes is 20?

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# 13.1 Independent Practice



**FL** 7.SP.3.7, 7.SP.3.7a



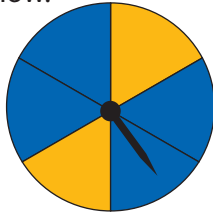
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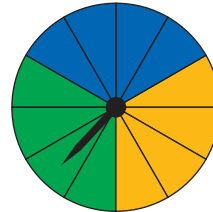
**Find the probability of each event. Write each answer as a fraction in simplest form, as a decimal to the nearest hundredth, and as a percent to the nearest whole number.**

- 8.** You spin the spinner shown. The spinner lands on yellow.



\_\_\_\_\_

- 9.** You spin the spinner shown. The spinner lands on blue or green.



\_\_\_\_\_

- 10.** A jar contains 4 cherry cough drops and 10 honey cough drops. You choose one cough drop without looking. The cough drop is cherry. \_\_\_\_\_
- 11.** You pick one card at random from a standard deck of 52 playing cards. You pick a black card. \_\_\_\_\_
- 12.** There are 12 pieces of fruit in a bowl. Five are lemons and the rest are limes. You choose a piece of fruit without looking. The piece of fruit is a lime. \_\_\_\_\_
- 13.** You choose a movie CD at random from a case containing 8 comedy CDs, 5 science fiction CDs, and 7 adventure CDs. The CD is **not** a comedy. \_\_\_\_\_
- 14.** You roll a number cube. You roll a number that is greater than 2 and less than 5. \_\_\_\_\_
- 15. Communicate Mathematical Ideas** The theoretical probability of a given event is  $\frac{9}{13}$ . Explain what each number represents.

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

- 16.** Leona has 4 nickels, 6 pennies, 4 dimes, and 2 quarters in a change purse. Leona lets her little sister Daisy pick a coin at random. If Daisy is equally likely to pick each type of coin, what is the probability that her coin is worth more than five cents? Explain.
- \_\_\_\_\_
- \_\_\_\_\_
- \_\_\_\_\_



**17. Critique Reasoning** A bowl of flower seeds contains 5 petunia seeds and 15 begonia seeds. Riley calculated the probability that a randomly selected seed is a petunia seed as  $\frac{1}{3}$ . Describe and correct Riley's error.

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**18.** There are 20 seventh graders and 15 eighth graders in a club. A club president will be chosen at random.

**a. Analyze Relationships** Compare the probabilities of choosing a seventh grader or an eighth grader.

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**b. Critical Thinking** If a student from one grade is more likely to be chosen than a student from the other, is the method unfair? Explain.

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A jar contains 8 red marbles, 10 blue ones, and 2 yellow ones. One marble is chosen at random. The color is recorded in the table, and then it is returned to the jar. This is repeated 40 times.

Red	Blue	Yellow
14	16	10

**19. Communicate Mathematical Ideas** Use proportional reasoning to explain how you know that for each color, the theoretical and experimental probabilities are not the same.

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**20. Persevere in Problem Solving** For which color marble is the experimental probability closest to the theoretical probability? Explain.

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# LESSON 13.2 Theoretical Probability of Compound Events

 **FL** 7.SP.3.8

Find probabilities of compound events using organized lists, tables, tree diagrams, ... 7.SP.3.8a, 7.SP.3.8b



## ESSENTIAL QUESTION

How do you find the probability of a compound event?

### EXPLORE ACTIVITY

 **FL** 7.SP.3.8, 7.SP.3.8a, 7.SP.3.8b

## Finding Probability Using a Table

Recall that a compound event consists of two or more simple events. To find the probability of a compound event, you write a ratio of the number of ways the compound event can happen to the total number of equally likely possible outcomes.



**Jacob rolls two fair number cubes. Find the probability that the sum of the numbers he rolls is 8.**

**STEP 1** Use the table to find the sample space for rolling a particular sum on two number cubes. Each cell is the sum of the first number in that row and column.

**STEP 2** How many possible outcomes are in the sample space? \_\_\_\_\_

**STEP 3** Circle the outcomes that give the sum of 8.

**STEP 4** How many ways are there to roll a sum of 8? \_\_\_\_\_

**STEP 5** What is the probability of rolling a sum of 8? \_\_\_\_\_

	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

### Reflect

1. Give an example of an event that is more likely than rolling a sum of 8.

\_\_\_\_\_

2. Give an example of an event that is less likely than rolling a sum of 8.

\_\_\_\_\_



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# Finding Probability Using a Tree Diagram

You can also use a tree diagram to calculate theoretical probabilities of compound events.

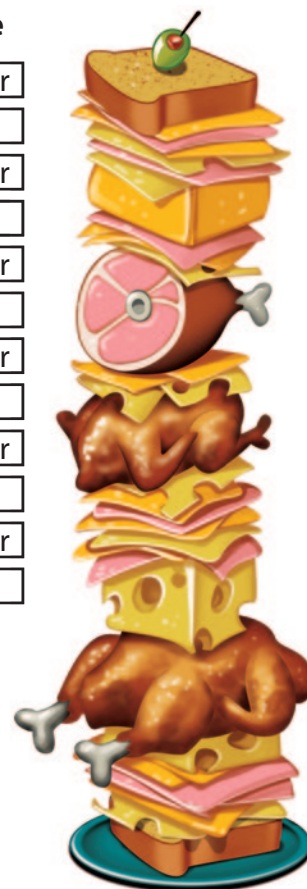
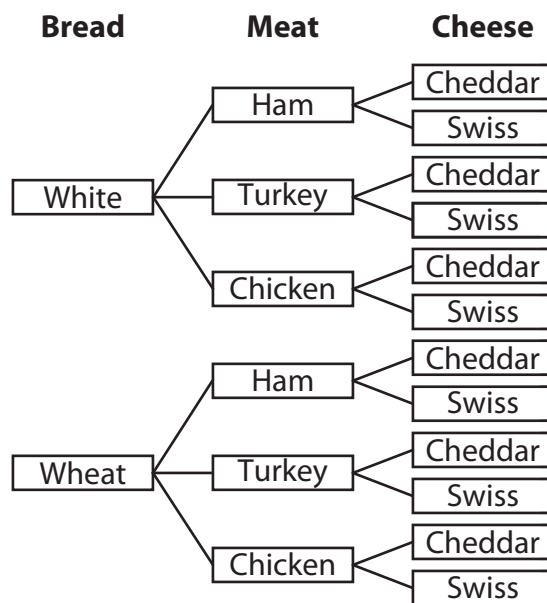
## EXAMPLE 1



FL 7.SP.3.8, 7.SP.3.8b

A deli prepares sandwiches with one type of bread (white or wheat), one type of meat (ham, turkey, or chicken), and one type of cheese (cheddar or Swiss). Each combination is equally likely. Find the probability of choosing a sandwich at random and getting turkey and Swiss on wheat bread.

**STEP 1** Make a tree diagram to find the sample space for the compound event.



### Math Talk

How many sandwich combinations are possible if one of the meat options is unavailable?

**STEP 2** Find the number of possible outcomes in the sample space: **12**

**STEP 3** Find the probability of choosing turkey and Swiss on wheat bread at random:  $\frac{1}{12}$

## YOUR TURN

Use the diagram from Example 1 to find the given probabilities.

- ham sandwich \_\_\_\_\_
- sandwich containing Swiss cheese \_\_\_\_\_



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# Finding Probability Using a List

One way to provide security for a locker or personal account is to assign it an access code number known only to the owner.



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## EXAMPLE 2



FL 7.SP.3.8, 7.SP.3.8b

The combination for Khiem's locker is a 3-digit code that uses the numbers 1, 2, and 3. Any of these numbers may be repeated. Find the probability that Khiem's randomly-assigned number is 222.

Make an organized list to find the sample space.

**STEP 1** List all the codes that start with 1 and have 1 as a second digit.

1	1	1
1	1	2
1	1	3

**STEP 2** List all the codes that start with 1 and have 2 as a second digit.

1	2	1
1	2	2
1	2	3

**STEP 3** List all the codes that start with 1 and have 3 as a second digit.

1	3	1
1	3	2
1	3	3

**STEP 4** You have now listed all the codes that start with 1. Repeat Steps 1–3 for codes that start with 2, and then for codes that start with 3.

2	1	1
2	1	2
2	1	3

2	2	1
2	2	2
2	2	3

2	3	1
2	3	2
2	3	3

3	1	1
3	1	2
3	1	3

3	2	1
3	2	2
3	2	3

3	3	1
3	3	2
3	3	3

**STEP 5** Find the number of outcomes in the sample space by counting all the possible codes. There are **27** such codes.

**STEP 6** Find the probability that Khiem's locker code is 222.

$$P(\text{Code } 222) = \frac{\text{number of favorable outcomes}}{\text{total number of possible outcomes}} = \frac{1}{27}$$

Notice that there are 3 possible first numbers, 3 possible second numbers, and 3 possible third numbers, or  $3 \times 3 \times 3 = 27$  numbers in all.

### Math Talk

Mathematical Practices



How could you find the probability that Khiem's locker code includes exactly two 1s?

## YOUR TURN

- Martha types a 4-digit code into a keypad to unlock her car doors. The code uses the numbers 1 and 0. If the digits are selected at random, what is the probability of getting a code with exactly two 0s? \_\_\_\_\_



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## Guided Practice

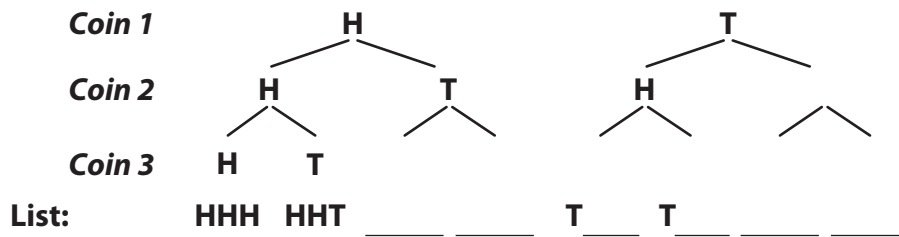
Drake rolls two fair number cubes. (Explore Activity)

- Complete the table to find the sample space for rolling a particular product on two number cubes.
- What is the probability that the product of the two numbers Drake rolls is a multiple of 4? \_\_\_\_\_
- What is the probability that the product of the two numbers Drake rolls is less than 13? \_\_\_\_\_

	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

You flip three coins and want to explore probabilities of certain events. (Examples 1 and 2)

- Complete the tree diagram and make a list to find the sample space.



- How many outcomes are in the sample space? \_\_\_\_\_
- List all the ways to get three tails. \_\_\_\_\_
- Complete the expression to find the probability of getting three tails.

$$P = \frac{\text{number of outcomes with } \boxed{\phantom{000}}}{\text{total number of possible outcomes}} = \frac{\boxed{\phantom{00}}}{\boxed{\phantom{000}}}$$

The probability of getting three tails when three coins are flipped is \_\_\_\_\_.

- What is the probability of getting exactly two heads?

There are \_\_\_\_\_ way(s) to obtain exactly two heads: HHT, \_\_\_\_\_

$$P = \frac{\text{number of outcomes with } \boxed{\phantom{000}}}{\text{total number of possible outcomes}} = \frac{\boxed{\phantom{00}}}{\boxed{\phantom{000}}}$$



### ESSENTIAL QUESTION CHECK-IN

- There are 6 ways a given compound event can occur. What else do you need to know to find the theoretical probability of the event?


\_\_\_\_\_



# 13.2 Independent Practice



**FL** 7.SP.3.8, 7.SP.3.8a, 7.SP.3.8b



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In Exercises 10–12, use the following information. Mattias gets dressed in the dark one morning and chooses his clothes at random. He chooses a shirt (green, red, or yellow), a pair of pants (black or blue), and a pair of shoes (checkered or red).

10. Use the space below to make a tree diagram to find the sample space.

11. What is the probability that Mattias picks an outfit at random that includes red shoes? \_\_\_\_\_
12. What is the probability that no part of Mattias’s outfit is red? \_\_\_\_\_
13. Rhee and Pamela are two of the five members of a band. Every week, the band picks two members at random to play on their own for five minutes. What is the probability that Rhee and Pamela are chosen this week? \_\_\_\_\_

14. Ben rolls two number cubes. What is the probability that the sum of the numbers he rolls is less than 6? \_\_\_\_\_

15. Nhan is getting dressed. He considers two different shirts, three pairs of pants, and three pairs of shoes. He chooses one of each of the articles at random. What is the probability that he will wear his jeans but not his sneakers?

Shirt	Pants	Shoes
collared	khakis	sneakers
T-shirt	jeans	flip-flops
	shorts	sandals

16. **Communicate Mathematical Ideas** A ski resort has 3 chair lifts, each with access to 6 ski trails. Explain how you can find the number of possible outcomes when choosing a chair lift and a ski trail without making a list, a tree diagram, or table.

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17. **Explain the Error** For breakfast, Sarah can choose eggs, granola or oatmeal as a main course, and orange juice or milk for a drink. Sarah says that the sample space for choosing one of each contains  $3^2 = 9$  outcomes. What is her error? Explain.

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- 18. Represent Real-World Problems** A new shoe comes in two colors, black or red, and in sizes from 5 to 12, including half sizes. If a pair of the shoes is chosen at random for a store display, what is the probability it will be red and size 9 or larger? \_\_\_\_\_



**FOCUS ON HIGHER ORDER THINKING**

Work Area

- 19. Analyze Relationships** At a diner, Sondra tells the server, "Give me one item from each column." Gretchen says, "Give me one main dish and a vegetable." Who has a greater probability of getting a meal that includes salmon? Explain.

Main Dish	Vegetable	Side
Pasta	Carrots	Tomato soup
Salmon	Peas	Tossed salad
Beef	Asparagus	
Pork	Sweet potato	

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- 20.** The digits 1 through 5 are used for a set of locker codes.
- a. Look for a Pattern** Suppose the digits cannot repeat. Find the number of possible two-digit codes and three-digit codes. Describe any pattern and use it to predict the number of possible five-digit codes.

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- b. Look for a Pattern** Repeat part **a**, but allow digits to repeat.

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- c. Justify Reasoning** Suppose that a gym plans to issue numbered locker codes by choosing the digits at random. Should the gym use codes in which the digits can repeat or not? Justify your reasoning.

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LESSON  
**13.3**

# Making Predictions with Theoretical Probability



**FL** 7.SP.3.6

... predict the approximate relative frequency given the probability. Also 7.SP.3.7a



## ESSENTIAL QUESTION

How do you make predictions using theoretical probability?

## Using Theoretical Probability to Make a Quantitative Prediction

You can make quantitative predictions based on theoretical probability just as you did with experimental probability earlier.

### EXAMPLE 1



**FL** 7.SP.3.6

- A** You roll a standard number cube 150 times. Predict how many times you will roll a 3 or a 4.

The probability of rolling a 3 or a 4 is  $\frac{2}{6} = \frac{1}{3}$ .

**Method 1: Set up a proportion.**

$$\frac{1}{3} = \frac{x}{150}$$

Write a proportion. 1 out of 3 is how many out of 150?

$$\frac{1}{3} = \frac{x}{150}$$

$$\frac{1}{3} = \frac{50}{150}$$

$$x = 50$$

Since 3 times 50 is 150, multiply 1 times 50 to find the value of x.

**Method 2: Set up an equation and solve.**

$p(\text{rolling a 3 or 4}) \cdot \text{Number of events} = \text{Prediction}$

$$\frac{1}{3} \cdot 150 = x$$

Multiply the probability by the total number of rolls.

$$50 = x$$

Solve for x.

You can expect to roll a 3 or a 4 about 50 times out of 150.



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My Notes



- B** Celia volunteers at her local animal shelter. She has an equally likely chance to be assigned to the dog, cat, bird, or reptile section. If she volunteers 24 times, about how many times should she expect to be assigned to the dog section?

Set up a proportion. The probability of being assigned to the dog section is  $\frac{1}{4}$ .

$$\frac{1}{4} = \frac{x}{24}$$

Write a proportion. 1 out of 4 is how many out of 24?

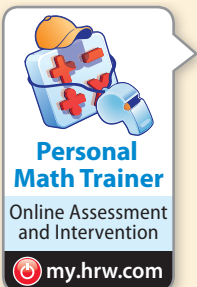
$$\frac{1}{4} = \frac{x}{24}$$

$$\frac{1}{4} = \frac{x}{24}$$

Since 4 times 6 is 24, multiply 1 times 6 to find the value of  $x$ .

$$x = 6$$

Celia can expect to be assigned to the dog section about 6 times out of 24.



### YOUR TURN

1. Predict how many times you will roll a number less than 5 if you roll a standard number cube 250 times.  
\_\_\_\_\_
2. You flip a fair coin 18 times. About how many times would you expect heads to appear?  
\_\_\_\_\_



## Using Theoretical Probability to Make a Qualitative Prediction

Earlier, you learned how to make predictions using experimental probability. You can use theoretical probabilities in the same way to help you predict or compare how likely events are.

## EXAMPLE 2



FL 7.SP.3.6, 7.SP.3.7a

- A** Herschel pulls a sock out of his drawer without looking and puts it on. The sock is black. There are 7 black socks, 8 white socks, and 5 striped socks left in the drawer. He pulls out a second sock without looking. Is it likely that he will be wearing matching socks to school?

Find the theoretical probability that Herschel picks a matching sock and the probability that he picks one that does not match.

$$P(\text{matching}) = \frac{7}{20}$$

$$P(\text{not matching}) = 1 - \frac{7}{20} = \frac{13}{20}$$

$$P(\text{not matching}) = 1 - P(\text{matching})$$

The probability that Herschel picks a matching sock is about half the probability that he picks one that does not match. It is likely that he will **not** be wearing matching socks to school.

- B** All 2,000 customers at a gym are randomly assigned a 3-digit security code that they use to access their online accounts. The codes are made up of the digits 0 through 4, and the digits can be repeated. Is it likely that fewer than 10 of the customers are issued the code 103?

Set up a proportion. The probability of the code 103 is  $\frac{1}{125}$ .

$$\frac{1}{125} = \frac{x}{2,000}$$

Write a proportion. 1 out of 125 is how many out of 2,000?

$$\frac{1}{125} = \frac{16}{2,000}$$

Since 125 times 16 is 2,000, multiply 1 times 16 to find the value of  $x$ .

There are 5 possible first numbers, 5 possible second numbers, and 5 possible third numbers. So, the probability of any one code is  $\frac{1}{5} \cdot \frac{1}{5} \cdot \frac{1}{5} = \frac{1}{125}$ .

It is **not** likely that fewer than 10 of the customers get the same code. It is more likely that 16 members get the code 103.

### YOUR TURN

- 3.** A bag of marbles contains 8 red marbles, 4 blue marbles, and 5 white marbles. Tom picks a marble at random. Is it more likely that he picks a red marble or a marble of another color?
- \_\_\_\_\_
- 4.** At a fundraiser, a school group charges \$6 for tickets for a “grab bag.” You choose one bill at random from a bag that contains 40 \$1 bills, 20 \$5 bills, 5 \$10 bills, 5 \$20 bills, and 1 \$100 bill. Is it likely that you will win enough to pay for your ticket? Justify your answer.
- \_\_\_\_\_
- \_\_\_\_\_
- \_\_\_\_\_

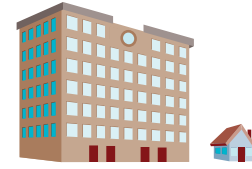


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## Guided Practice



1. Bob works at a construction company. He has an equally likely chance to be assigned to work different crews every day. He can be assigned to work on crews building apartments, condominiums, or houses. If he works 18 days a month, about how many times should he expect to be assigned to the house crew? (Example 1)

**STEP 1** Find the probabilities of being assigned to each crew.

Apartment  Condo  House

The probability of being assigned to the house crew is \_\_\_\_\_

**STEP 2** Set up and solve a proportion.

$$\frac{\boxed{\phantom{00}}}{\boxed{\phantom{00}}} = \frac{x}{\boxed{\phantom{00}}} \quad x = \underline{\hspace{2cm}}$$

Bob can expect to be assigned to the house crew about \_\_\_\_\_ times out of 18.

2. During a raffle drawing, half of the ticket holders will receive a prize. The winners are equally likely to win one of three prizes: a book, a gift certificate to a restaurant, or a movie ticket. If there are 300 ticket holders, predict the number of people who will win a movie ticket. (Example 1) \_\_\_\_\_
3. In Mr. Jawarani's first period math class, there are 9 students with hazel eyes, 10 students with brown eyes, 7 students with blue eyes, and 2 students with green eyes. Mr. Jawarani picks a student at random. Which color eyes is the student most likely to have? Explain. (Example 2)

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### ESSENTIAL QUESTION CHECK-IN

4. How do you make predictions using theoretical probability?

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# 13.3 Independent Practice



**FL** 7.SP.3.6, 7.SP.3.7a



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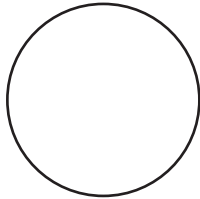
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**5.** A bag contains 6 red marbles, 2 white marbles, and 1 gray marble. You randomly pick out a marble, record its color, and put it back in the bag. You repeat this process 45 times. How many white or gray marbles do you expect to get?

\_\_\_\_\_

**6.** Using the blank circle below, draw a spinner with 8 equal sections and 3 colors—red, green, and yellow. The spinner should be such that you are equally likely to land on green or yellow, but more likely to land on red than either on green or yellow.



**Use the following for Exercises 7–9.**  
**In a standard 52-card deck, half of the cards are red and half are black. The 52 cards are divided evenly into 4 suits: spades, hearts, diamonds, and clubs. Each suit has three face cards (jack, queen, king), and an ace. Each suit also has 9 cards numbered from 2 to 10.**

**7.** Dawn draws 1 card, replaces it, and draws another card. Is it more likely that she draws 2 red cards or 2 face cards?

\_\_\_\_\_

\_\_\_\_\_

**8.** Luis draws 1 card from a deck, 39 times. Predict how many times he draws an ace.

\_\_\_\_\_

**9.** Suppose a solitaire player has played 1,000 games. Predict how many times the player turned over a red card as the first card.

\_\_\_\_\_

**10.** John and O’Neal are playing a board game in which they roll two number cubes. John needs to get a sum of 8 on the number cubes to win. O’Neal needs a sum of 11. If they take turns rolling the number cube, who is more likely to win? Explain.

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

**11.** Every day, Navya’s teacher randomly picks a number from 1 to 20 to be the number of the day. The number of the day can be repeated. There are 180 days in the school year. Predict how many days the number of the day will be greater than 15. \_\_\_\_\_

**12.** Eben rolls two standard number cubes 36 times. Predict how many times he will roll a sum of 4. \_\_\_\_\_

**13. Communicate Mathematical Ideas** Can you always show that a prediction based on theoretical probability is true by performing the event often enough? If so, explain why. If not, describe a situation that justifies your response.

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

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14. **Represent Real-World Problems** Give a real-world example of an experiment in which all of the outcomes are not equally likely. Can you make a prediction for this experiment, using theoretical probability?

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**FOCUS ON HIGHER ORDER THINKING**

15. **Critical Thinking** Pierre asks Sherry a question involving the theoretical probability of a compound event in which you flip a coin and draw a marble from a bag of marbles. The bag of marbles contains 3 white marbles, 8 green marbles, and 9 black marbles. Sherry's answer, which is correct, is  $\frac{12}{40}$ . What was Pierre's question?

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16. **Make a Prediction** Horace is going to roll a standard number cube and flip a coin. He wonders if it is more likely that he rolls a 5 **and** the coin lands on heads, or that he rolls a 5 **or** the coin lands on heads. Which event do you think is more likely to happen? Find the probability of both events to justify or reject your initial prediction.

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17. **Communicate Mathematical Ideas** Cecil solved a theoretical prediction problem and got this answer: "The spinner will land on the red section 4.5 times." Is it possible to have a prediction that is not a whole number? If so, give an example.

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Work Area



# LESSON 13.4 Using Technology to Conduct a Simulation

 **FL** 7.SP.3.8c

Design and use a simulation to generate frequencies for compound events. *Also 7.SP.3.8*



## ESSENTIAL QUESTION

How can you use technology simulations to estimate probabilities?

## Designing and Conducting a Simulation for a Simple Event

You can use a graphing calculator or computer to generate random numbers and conduct a simulation.



### EXAMPLE 1



 **FL** 7.SP.3.8c

A cereal company is having a contest. There are codes for winning prizes in 30% of its cereal boxes. Find an experimental probability that you have to buy *exactly* 3 boxes of cereal before you find a winning code.

**STEP 1** Choose a model.

The probability of finding a winning code is  $30\% = \frac{3}{10}$ .

Use whole numbers from 1 to 10.

Let three numbers represent buying a box with a winning code.

Winning code: 1, 2, 3    Nonwinning code: 4, 5, 6, 7, 8, 9, 10

**STEP 2** Generate random numbers from 1 to 10 until you get one that represents a box with a winning code. Record how many boxes you bought before finding a winning code.

5 numbers generated: 9, 6, 7, 8, 1

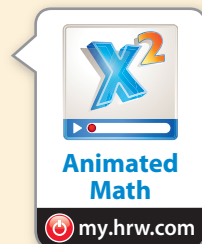
1 represents a box with a winning code.

**STEP 3** Perform multiple trials by repeating Step 2.


**STEP 4** Find the experimental probability.

In 1 of 10 trials, you bought exactly 3 boxes of cereal before finding a winning code. The experimental probability is  $\frac{1}{10}$ , or 10%.

Trial	Numbers generated	Boxes bought
1	9, 6, 7, 8, 1	5
2	2	1
3	10, 4, 8, 1	4
4	4, 10, 7, 1	4
5	2	1
6	4, 3	2
7	3	1
8	7, 5, 2	3
9	8, 5, 4, 8, 10, 3	6
10	9, 1	2



Trial 8 represents a winning code after buying 3 boxes.



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## YOUR TURN

- An elephant has a 50% chance of giving birth to a male or a female calf. Use a simulation to find an experimental probability that the elephant gives birth to 3 male calves before having a female calf. (*Hint: Use 0s and 1s. Let 0 represent a male calf, and 1 represent a female calf. Generate random numbers until you get a 1.*)

Trial	Numbers generated	3 Males first
1		
2		
3		
4		
5		

Trial	Numbers generated	3 Males first
6		
7		
8		
9		
10		

### Math Talk

Mathematical Practices

Could you generate random numbers from a list of more than 2 numbers? Explain.



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## Designing and Conducting a Simulation for a Compound Event

You can use random numbers to simulate compound events as well as simple events.

### EXAMPLE 2



FL 7.SP.3.8c, 7.SP.3.8

Suppose that there is a 20% chance that a particular volcano will erupt in any given decade. Find an experimental probability that the volcano will erupt in at least 1 of the next 5 decades.

**STEP 1** Choose a model.

The probability of an eruption is  $20\% = \frac{1}{5}$ .  
Use whole numbers from 1 to 5.

Let 1 represent a decade with an eruption.

Let 2, 3, 4, and 5 represent a decade without an eruption.



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**STEP 2** Generate 5 random numbers from 1 to 5. Record the number of decades with an eruption.

5 numbers generated: 3, 1, 3, 4, 2      Eruption decades: 1

**STEP 3** Perform multiple trials by repeating Step 2. Calculate the percent of trials in which there was an eruption in at least 1 of the 5 decades.

Trial	Numbers generated	Eruption decades
1	3, 1, 3, 4, 2	1
2	3, 2, 2, 4, 5	0
3	1, 3, 3, 2, 5	1
4	5, 3, 4, 5, 4	0
5	5, 5, 3, 2, 4	0

Trial	Numbers generated	Eruption decades
6	2, 3, 3, 4, 2	0
7	1, 2, 4, 1, 4	2
8	1, 3, 2, 1, 5	2
9	1, 2, 4, 2, 5	1
10	5, 5, 3, 2, 4	0

In 5 out of the 10 trials, there was an eruption in at least 1 of the 5 decades. The experimental probability of an eruption in at least 1 of the next 5 decades is  $\frac{5}{10} = 50\%$ .

## YOUR TURN

2. Matt guesses the answers on a quiz with 5 true-false questions. The probability of guessing a correct answer on each question is 50%. Use a simulation to find an experimental probability that he gets at least 2 questions right. (*Hint:* Use 0s and 1s. Let 0s represent incorrect answers, and 1s represent correct answers. Perform 10 trials, generating 5 random numbers in each, and count the number of 1s.)

Trial	Numbers generated	Correct answers
1		
2		
3		
4		
5		

Trial	Numbers generated	Correct answers
6		
7		
8		
9		
10		

## Guided Practice

There is a 30% chance that T'Shana's county will have a drought during any given year. She performs a simulation to find the experimental probability of a drought in at least 1 of the next 4 years. (Examples 1 and 2)

1. T'Shana's model involves the whole numbers from 1 to 10. Complete the description of her model.

Let the numbers 1 to 3 represent

and the numbers 4 to 10 represent

Perform multiple trials, generating  random numbers each time.

2. Suppose T'Shana used the model described in Exercise 1 and got the results shown in the table. Complete the table.

Trial	Numbers generated	Drought years
1	10, 3, 5, 1	
2	10, 4, 6, 5	
3	3, 2, 10, 3	
4	2, 10, 4, 4	
5	7, 3, 6, 3	

Trial	Numbers generated	Drought years
6	8, 4, 8, 5	
7	6, 2, 2, 8	
8	6, 5, 2, 4	
9	2, 2, 3, 2	
10	6, 3, 1, 5	

3. According to the simulation, what is the experimental probability that there will be a drought in the county in at least 1 of the next 4 years? \_\_\_\_\_



### ESSENTIAL QUESTION CHECK-IN

4. You want to generate random numbers to simulate an event with a 75% chance of occurring. Describe a model you could use.

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# 13.4 Independent Practice



**FL** 7.SP.3.8, 7.SP.3.8c



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Every contestant on a game show has a 40% chance of winning. In the simulation below, the numbers 1–4 represent a winner, and the numbers 5–10 represent a nonwinner. Numbers were generated until one that represented a winner was produced.

Trial	Numbers generated
1	7, 4
2	6, 5, 2
3	1
4	9, 1
5	3

Trial	Numbers generated
6	8, 8, 6, 2
7	2
8	5, 9, 4
9	10, 3
10	1

- In how many of the trials did it take exactly 4 contestants to get a winner? \_\_\_\_\_
- Based on the simulation, what is the experimental probability that it will take exactly 4 contestants to get a winner? \_\_\_\_\_

Over a 100-year period, the probability that a hurricane struck Rob’s city in any given year was 20%. Rob performed a simulation to find an experimental probability that a hurricane would strike the city in at least 4 of the next 10 years. In Rob’s simulation, 1 represents a year with a hurricane.

Trial	Numbers generated
1	2, 5, 3, 2, 5, 5, 1, 4, 5, 2
2	1, 1, 5, 2, 2, 1, 3, 1, 1, 5
3	4, 5, 4, 5, 5, 4, 3, 5, 1, 1
4	1, 5, 5, 5, 1, 2, 2, 3, 5, 3
5	5, 1, 5, 3, 5, 3, 4, 5, 3, 2

Trial	Numbers generated
6	1, 1, 5, 5, 1, 4, 2, 2, 3, 4
7	2, 1, 5, 3, 1, 5, 1, 2, 1, 4
8	2, 4, 3, 2, 4, 4, 2, 1, 3, 1
9	3, 2, 1, 4, 5, 3, 5, 5, 1, 2
10	3, 4, 2, 4, 3, 5, 2, 3, 5, 1

- According to Rob’s simulation, what was the experimental probability that a hurricane would strike the city in at least 4 of the next 10 years? \_\_\_\_\_
- Analyze Relationships** Suppose that over the 10 years following Rob’s simulation, there was actually 1 year in which a hurricane struck. How did this compare to the results of Rob’s simulation?  
 \_\_\_\_\_  
 \_\_\_\_\_

9. **Communicate Mathematical Ideas** You generate three random whole numbers from 1 to 10. Do you think that it is unlikely or even impossible that all of the numbers could be 10? Explain?

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10. Erika collects baseball cards, and 60% of the packs contain a player from her favorite team. Use a simulation to find an experimental probability that she has to buy exactly 2 packs before she gets a player from her favorite team.

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**H.O.T.** FOCUS ON HIGHER ORDER THINKING

11. **Represent Real-World Problems** When Kate plays basketball, she usually makes 37.5% of her shots. Design and conduct a simulation to find the experimental probability that she makes at least 3 of her next 10 shots. Justify the model for your simulation.

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12. **Justify Reasoning** George and Susannah used a simulation to simulate the flipping of 8 coins 50 times. In all of the trials, at least 5 heads came up. What can you say about their simulation? Explain.

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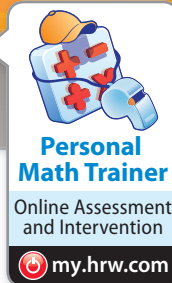
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Work Area

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# Ready to Go On?



## 13.1, 13.2 Theoretical Probability of Simple and Compound Events

Find the probability of each event. Write your answer as a fraction, as a decimal, and as a percent.

- You choose a marble at random from a bag containing 12 red, 12 blue, 15 green, 9 yellow, and 12 black marbles. The marble is red. \_\_\_\_\_
- You draw a card at random from a shuffled deck of 52 cards. The deck has four 13-card suits (diamonds, hearts, clubs, spades). The card is a diamond or a spade. \_\_\_\_\_

## 13.3 Making Predictions with Theoretical Probability

- A bag contains 23 red marbles, 25 green marbles, and 18 blue marbles. You choose a marble at random from the bag. What color marble will you most likely choose? \_\_\_\_\_

## 13.4 Using Technology to Conduct a Simulation

- Bay City has a 25% chance of having a flood in any given decade. The table shows the results of a simulation using random numbers to find the experimental probability that there will be a flood in Bay City in at least 1 of the next 5 decades. In the table, the number 1 represents a decade with a flood. The numbers 2 through 5 represent a decade without a flood.

Trial	Numbers generated	Trial	Numbers generated
1	2, 2, 5, 5, 5	6	4, 2, 2, 5, 4
2	3, 2, 3, 5, 4	7	1, 3, 2, 4, 4
3	5, 5, 5, 4, 3	8	3, 5, 5, 2, 1
4	5, 1, 3, 3, 5	9	4, 3, 3, 2, 5
5	4, 5, 5, 3, 2	10	5, 4, 1, 2, 1

According to the simulation, what is the experimental probability of a flood in Bay City in at least 1 of the next 5 decades? \_\_\_\_\_

### ESSENTIAL QUESTION

- How can you use theoretical probability to make predictions in real-world situations?

\_\_\_\_\_

\_\_\_\_\_



## Selected Response

- What is the probability of flipping two fair coins and having both show tails?
 

(A)  $\frac{1}{8}$                       (C)  $\frac{1}{3}$

(B)  $\frac{1}{4}$                       (D)  $\frac{1}{2}$
- A bag contains 8 white marbles and 2 black marbles. You pick out a marble, record its color, and put the marble back in the bag. If you repeat this process 45 times, how many times would you expect to remove a white marble from the bag?
 

(A) 9                      (C) 36

(B) 32                      (D) 40
- Philip rolls a standard number cube 24 times. Which is the best prediction for the number of times he will roll a number that is even and less than 4?
 

(A) 2                      (C) 4

(B) 3                      (D) 6
- A set of cards includes 24 yellow cards, 18 green cards, and 18 blue cards. What is the probability that a card chosen at random is **not** green?
 

(A)  $\frac{3}{10}$                       (C)  $\frac{3}{5}$

(B)  $\frac{4}{10}$                       (D)  $\frac{7}{10}$
- A rectangle made of square tiles measures 10 tiles long and 8 tiles wide. What is the width of a similar rectangle whose length is 15 tiles?
 

(A) 3 tiles                      (C) 13 tiles

(B) 12 tiles                      (D) 18.75 tiles

- The Fernandez family drove 273 miles in 5.25 hours. How far would they have driven at that rate in 4 hours?
 

(A) 208 miles                      (C) 280 miles

(B) 220 miles                      (D) 358 miles
- There are 20 tennis balls in a bag. Five are orange, 7 are white, 2 are yellow, and 6 are green. You choose one at random. Which color ball are you **least** likely to choose?
 

(A) green                      (C) white

(B) orange                      (D) yellow

## Mini-Task

- Center County has had a 1 in 6 (or about 16.7%) chance of a tornado in any given decade. In a simulation to consider the probability of tornadoes in the next 5 decades, Ava rolled a number cube. She let a 1 represent a decade with a tornado, and 2–6 represent decades without tornadoes. What experimental probability did Ava find for each event?

Trial	Numbers Generated	Trial	Numbers Generated
1	2, 2, 3, 1, 5	6	4, 5, 2, 2, 4
2	3, 5, 6, 4, 5	7	5, 1, 6, 3, 1
3	1, 3, 3, 2, 2	8	1, 2, 1, 2, 4
4	6, 3, 3, 5, 4	9	1, 4, 4, 1, 4
5	4, 1, 4, 4, 4	10	3, 6, 5, 3, 6

- That Center County has a tornado in at least one of the next five decades.  
\_\_\_\_\_
- That Center County has a tornado in exactly one of the next five decades.  
\_\_\_\_\_



## MODULE 12 Experimental Probability



### ESSENTIAL QUESTION

How can you use experimental probability to solve real-world problems?

### EXAMPLE 1

What is the probability of picking a red marble from a jar with 5 green marbles and 2 red marbles?

$$\begin{aligned}
 P(\text{picking a red marble}) &= \frac{\text{number of red marbles}}{\text{number of total marbles}} \\
 &= \frac{2}{7} \quad \begin{array}{l} \text{There are 2 red marbles.} \\ \text{The total number of marbles is } 2 + 5 = 7. \end{array}
 \end{aligned}$$

### EXAMPLE 2

For one month, a doctor recorded information about new patients as shown in the table.

	Senior	Adult	Young adult	Child
Female	5	8	2	14
Male	3	10	1	17

What is the experimental probability that his next new patient is a female adult?

$$\begin{aligned}
 P(\text{new patient is a female adult}) &= \frac{\text{number of female adults}}{\text{total number of patients}} \\
 P &= \frac{8}{60} = \frac{2}{15}
 \end{aligned}$$

What is the experimental probability that his next new patient is a child?

$$\begin{aligned}
 P(\text{new patient is a child}) &= \frac{\text{number of children}}{\text{total number of patients}} \\
 P &= \frac{31}{60}
 \end{aligned}$$

### EXERCISES

Find the probability of each event. (Lesson 12.1)

1. Rolling a 5 on a fair number cube.

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2. Picking a 7 from a standard deck of 52 cards. A standard deck includes 4 cards of each number from 2 to 10.

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3. Picking a blue marble from a bag of 4 red marbles, 6 blue marbles, and 1 white marble.

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4. Rolling a number greater than 7 on a 12-sided number cube.

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### Key Vocabulary

- complement (*complemento*)
- compound event (*suceso compuesto*)
- event (*suceso*)
- experiment (*experimento*)
- experimental probability (*probabilidad experimental*)
- outcome (*resultado*)
- probability (*probabilidad*)
- sample space (*espacio muestral*)
- simple event (*suceso simple*)
- simulation (*simulación*)
- trial (*prueba*)

5. Christopher picked coins randomly from his piggy bank and got the numbers of coins shown in the table. Find each experimental probability. (Lessons 12.2, 12.3)

Penny	Nickel	Dime	Quarter
7	2	8	6

- a. The next coin that Christopher picks is a quarter. \_\_\_\_\_
- b. The next coin that Christopher picks is not a quarter. \_\_\_\_\_
- c. The next coin that Christopher picks is a penny or a nickel. \_\_\_\_\_
6. A grocery store manager found that 54% of customers usually bring their own bags. In one afternoon, 82 out of 124 customers brought their own grocery bags. Did a greater or lesser number of people than usual bring their own bags? (Lesson 12.4)

## MODULE 13 Theoretical Probability

### ESSENTIAL QUESTION

How can you use theoretical probability to solve real-world problems?

**Key Vocabulary**  
theoretical probability  
(probabilidad teórica)

### EXAMPLE 1

- A. Lola rolls two fair number cubes. What is the probability that the two numbers Lola rolls include at least one 4 and have a product of at least 16?**

There are 5 pairs of numbers that include a 4 and have a product of at least 16:

(4, 4), (4, 5), (4, 6), (5, 4), (6, 4)

Find the probability.

$$P = \frac{\text{number of possible ways}}{\text{total number of possible outcomes}} = \frac{5}{36}$$

	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

- B. Suppose Lola rolls the two number cubes 180 times. Predict how many times she will roll two numbers that include a pair of numbers like the ones described above.**

One way to answer is to write and solve an equation.

$$\frac{5}{36} \times 180 = x \quad \text{Multiply the probability by the total number of rolls.}$$

$$25 = x \quad \text{Solve for } x.$$

Lola can expect to roll two numbers that include at least one 4 and have a product of 16 or more about 25 times.

## EXAMPLE 2

A store has a sale bin of soup cans. There are 6 cans of chicken noodle soup, 8 cans of split pea soup, 8 cans of minestrone, and 13 cans of vegetable soup. Find the probability of picking each type of soup at random. Then predict what kind of soup a customer is most likely to pick.

$$P(\text{chicken noodle}) = \frac{6}{35}$$

$$P(\text{split pea}) = \frac{8}{35}$$

$$P(\text{minestrone}) = \frac{8}{35}$$

$$P(\text{vegetable}) = \frac{13}{35}$$

The customer is most likely to pick vegetable soup. That is the event that has the greatest probability.

## EXERCISES

Find the probability of each event. (Lessons 13.1, 13.2)

1. Graciela picks a white mouse at random from a bin of 8 white mice, 2 gray mice, and 2 brown mice.  
\_\_\_\_\_
2. Theo spins a spinner that has 12 equal sections marked 1 through 12. It does **not** land on 1.  
\_\_\_\_\_
3. Tania flips a coin three times. The coin lands on heads twice and on tails once, not necessarily in that order.  
\_\_\_\_\_
4. Students are randomly assigned two-digit codes. Each digit is either 1, 2, 3, or 4. Guy is given the number 11.  
\_\_\_\_\_
5. Patty tosses a coin and rolls a number cube. (Lesson 13.3)
  - a. Find the probability that the coin lands on heads and the cube lands on an even number.  
\_\_\_\_\_
  - b. Patty tosses the coin and rolls the number cube 60 times. Predict how many times the coin will land on heads and the cube will land on an even number.  
\_\_\_\_\_
6. Rajan's school is having a raffle. The school sold raffle tickets with 3-digit numbers. Each digit is either 1, 2, or 3. The school also sold 2 tickets with the number 000. Which number is more likely to be picked, 123 or 000? (Lesson 13.3)  
\_\_\_\_\_

7. Suppose you know that over the last 10 years, the probability that your town would have at least one major storm was 40%. Describe a simulation that you could use to find the experimental probability that your town will have at least one major storm in at least 3 of the next 5 years. (Lesson 13.4)

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## Unit 6 Performance Tasks

1. **CAREERS IN MATH** **Meteorologist** A meteorologist predicts a 20% chance of rain for the next two nights, and a 75% chance of rain on the third night.

- a. On which night is it most likely to rain? On that night, is it *likely* to rain or *unlikely* to rain?

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- b. Tara would like to go camping for the next 3 nights, but will not go if it is likely to rain on all 3 nights. Should she go? Use probability to justify your answer.

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2. Sinead tossed 4 coins at the same time. She did this 50 times, and 6 of those times, all 4 coins showed the same result (heads or tails).

- a. Find the experimental probability that all 4 coins show the same result when tossed.

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- b. Can you determine the experimental probability that **no** coin shows heads? Explain.

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- c. Suppose Sinead tosses the coins 125 more times. Use experimental probability to predict the number of times that all 4 coins will show heads or tails. Show your work.

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## Selected Response

- A pizza parlor offers thin, thick, and traditional style pizza crusts. You can get pepperoni, beef, mushrooms, olives, or peppers for toppings. You order a one-topping pizza. How many outcomes are in the sample space?
 

(A) 3                                      (C) 8  
(B) 5                                      (D) 15
- A bag contains 9 purple marbles, 2 blue marbles, and 4 pink marbles. The probability of randomly drawing a blue marble is  $\frac{2}{15}$ . What is the probability of **not** drawing a blue marble?
 

(A)  $\frac{2}{15}$                                       (C)  $\frac{11}{15}$   
(B)  $\frac{4}{15}$                                       (D)  $\frac{13}{15}$
- During the month of April, Dora kept track of the bugs she saw in her garden. She saw a ladybug on 23 days of the month. What is the experimental probability that she will see a ladybug on May 1?
 

(A)  $\frac{1}{23}$                                       (C)  $\frac{1}{2}$   
(B)  $\frac{7}{30}$                                       (D)  $\frac{23}{30}$
- Ryan flips a coin 8 times and gets tails all 8 times. What is the experimental probability that Ryan will get heads the next time he flips the coin?
 

(A) 1    (C)  $\frac{1}{8}$   
(B)  $\frac{1}{2}$     (D) 0
- A used guitar is on sale for \$280. Derek offers the seller  $\frac{3}{4}$  of the advertised price. How much does Derek offer for the guitar?
 

(A) \$180                                      (C) \$240  
(B) \$210                                      (D) \$270

- Jay tossed two coins several times and then recorded the results in the table below.

Coin Toss Results				
Toss 1	Toss 2	Toss 3	Toss 4	Toss 5
H; H	H; T	T; H	T; T	T; H

What is the experimental probability that the coins will land on different sides on his next toss?

- (A)  $\frac{1}{5}$     (C)  $\frac{3}{5}$   
(B)  $\frac{2}{5}$     (D)  $\frac{4}{5}$
- What is the probability of tossing two fair coins and having exactly one land tails side up?
 

(A)  $\frac{1}{8}$     (C)  $\frac{1}{3}$   
(B)  $\frac{1}{4}$     (D)  $\frac{1}{2}$
  - Find the percent change from 60 to 96.
 

(A) 37.5% decrease  
(B) 37.5% increase  
(C) 60% decrease  
(D) 60% increase
  - A bag contains 6 white beads and 4 black beads. You pick out a bead, record its color, and put the bead back in the bag. You repeat this process 35 times. Which is the best prediction of how many times you would expect to remove a white bead from the bag?
 

(A) 6    (C) 18  
(B) 10    (D) 21
  - A set of cards includes 20 yellow cards, 16 green cards, and 24 blue cards. What is the probability that a blue card is chosen at random?
 

(A) 0.04    (C) 0.4  
(B) 0.24    (D) 0.66

11. Jason, Erik, and Jamie are friends in art class. The teacher randomly chooses 2 of the 21 students in the class to work together on a project. What is the probability that two of these three friends will be chosen?

- (A)  $\frac{1}{105}$   
 (B)  $\frac{1}{70}$   
 (C)  $\frac{34}{140}$   
 (D)  $\frac{4}{50}$

12. Philip rolls a number cube 12 times. Which is the best prediction for the number of times that he will roll a number that is odd and less than 5?

- (A) 2                      (C) 4  
 (B) 3                      (D) 6



**Estimate your answer before solving the problem. Use your estimate to check the reasonableness of your answer.**

13. A survey reveals that one airline's flights have a 92% probability of being on time. Based on this, out of 4000 flights in a year, how many flights would you predict will arrive on time?

- (A) 368                      (C) 3,680  
 (B) 386                      (D) 3,860

14. Matt's house number is a two-digit number. Neither of the digits is 0 and the house number is even. What is the probability that Matt's house number is 18?

- (A)  $\frac{1}{45}$                       (C)  $\frac{1}{18}$   
 (B)  $\frac{1}{36}$                       (D)  $\frac{1}{16}$

## Mini-Tasks

15. Laura picked a crayon randomly from a box, recorded the color, and then placed it back in the box. She repeated the process and recorded the results in the table.

Red	Blue	Yellow	Green
5	6	7	2

Find each experimental probability. Write your answers in simplest form.

- a. The next crayon Laura picks is red.

\_\_\_\_\_

- b. The next crayon Laura picks is **not** red.

\_\_\_\_\_

16. For breakfast, Trevor has a choice of 3 types of bagels (plain, sesame, or multigrain), 2 types of eggs (scrambled or poached), and 2 juices (orange or apple).

- a. Use the space below to make a tree diagram to find the sample space.

- b. If he chooses at random, what is the probability that Trevor eats a breakfast that has orange juice?

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